

DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE, PERAMBALUR - 621212

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EC6405 - CONTROL SYSTEM ENGINEERING

QUESTION BANK

UNIT - I CONTROL SYSTEM MODELING

PART - A

1. What is control system? (Nov/Dec-16)

A system consists of a number of components connected together to perform a specific function. In a system when the output quantity is controlled by varying the input quantity then the system is called control system.

2. Distinguish between open loop and closed loop system. (Apr/May-17, May/Jun-16, Nov/Dec-14, Nov/Dec-10, May/Jun-10)

S. No	Open Loop	Closed Loop
1	Inaccurate	Accurate
2	Simple and Economical	Complex and Costlier
3	The change in output due to external disturbance are not corrected	The change in output due to external disturbance are corrected automatically
4	May oscillate and become unstable	They are generally stable

3. Define open loop control system. (Nov/Dec-11, May/Jun-11)

The control system in which the output quantity has no effect upon the input quantity are called open loop control system. This means that the output is not feedback to the input for correction.

4. Define closed loop control system. (Nov/Dec-11, May/Jun-11)

The control system in which the output has an effect upon the input quantity so as to maintain the desired output values are called closed loop control system.

5. What are the advantages of closed loop control system? (Nov/Dec-15, Nov/Dec-12, May/Jun-12)

The negative feedback results in better stability in steady state and rejects any disturbance signals.

6. Define transfer function. (Nov/Dec-13, Nov/Dec-10)

The T.F of a system is defined as the ratio of the Laplace transform of output (Y(s)) to Laplace transform of input (X(s)) with zero initial conditions.

$$H(s) = Y(s) / X(s)$$

7. What are the basic elements used for modeling mechanical translational system? Write the force balance equation of basic elements. (Nov/Dec-16, Apr/May-15, Nov/Dec-15)

✓ **Mechanical translational system**

Mass, spring and dashpot.

$$F = M d^2x / dt^2$$

$$F = B dx / dt$$

$$F = Kx$$

8. What are the basic elements used for modeling mechanical rotational system? Write the force balance equation of basic elements. (Nov/Dec-16, Apr/May-15, Nov/Dec-15)

✓ **Mechanical rotational system**

Moment of inertia J, Dashpot with rotational frictional coefficient B & Torsion spring with stiffness K

$$F = J d^2 \theta / dt^2$$

$$F = B d\theta / dt$$

$$F = K\theta$$

9. What is Block Diagram? What are the basic components of Block diagram? (Nov/Dec-15, Nov/Dec-11)

A Block Diagram of a system is a pictorial representation of the functions performed by each component of the system and shows the flow of signals.

The basic elements of block diagram are blocks, branch point and summing point.

10. What is a signal flow graph and Write Masons Gain formula. (Apr/May-17, May/Jun-16, May/Jun-15, Nov/Dec-14, May/Jun-14, May/Jun-13)

A signal flow graph is a diagram that represents a set of simultaneous algebraic equations. By taking L.T the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain.

Masons Gain formula states that the overall gain of the system is $T = 1 / \Delta k P_k$

k- No.of forward paths in the signal flow graph.

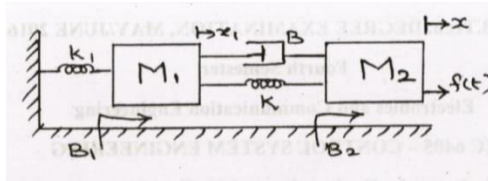
P_k- Forward path gain of kth forward path

$\Delta k = 1 - [\text{sum of individual loop gains}] + [\text{sum of gain products of all possible combinations of two non touching loops}] - [\text{sum of gain products of all possible combinations of three non touching loops}] + \dots$

k - for that part of the graph which is not touching kth forward path.

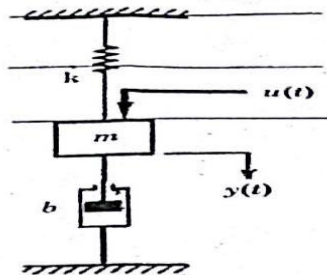
PART-B (Answers as Hint)

1. Write the differential equations governing the mechanical translational system as shown in figure and determine the transfer function. (May/Jun-16)

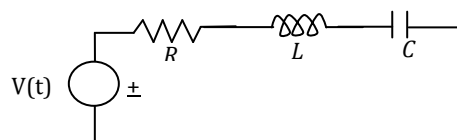


- To find equation on M_1 and M_2 . (5 Marks)
 - ✓ $M_1 \frac{d^2x_1}{dt^2} + B \frac{d(x_1-x)}{dt} + B_1 \frac{dx_1}{dt} + K_1x + K(x_1 - x) = 0$
 - ✓ $M_2 \frac{d^2x}{dt^2} + B \frac{d(x-x_1)}{dt} + B_1 \frac{dx}{dt} + K(x - x_1) = f(t)$
- To find Laplace transform equation for above equation. (5 Marks)
 - ✓ $M_1S^2X_1(S) + BS[X_1(S) - X(S)] + B_1X_1(S) + K_1X(S) + K[X_1(S) - X(S)] = 0$
 - ✓ $M_2S^2X(S) + BS[X(S) - X_1(S)] + B_1X(S) + K[X(S) - X_1(S)] = F(S)$
- Solve the above equation and find transfer function. (6 Marks)
 - ✓ Transfer function = $\frac{X(S)}{F(S)} = \frac{M_1S^2 + K_1 + K + S[B_1 + B]}{[M_2S^2 + S(B + B_2) + K_2][M_1S^2 + K_1 + K + S(B_1 + B)] - [BS + K]^2}$

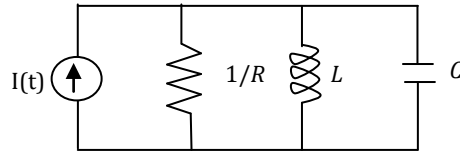
2. Draw the equivalent electrical analogous circuit for the mechanical system in Fig using force-voltage analogy. (Apr/May-17)



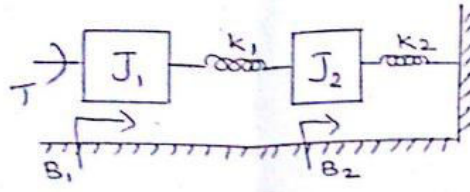
- To find equation on M_1 and M_2 . (5 Marks)
 - ✓ $M \frac{d^2y(t)}{dt^2} + B \frac{dy(t)}{dt} + Ky(t) = u(t)$
- Force voltage and force Current equation. (5 Marks)
 - (i) Force voltage equation:
 - ✓ $V(t) = L \frac{di}{dt} + \frac{1}{C} \int idt + Ri$
 - (ii) Force current equation:
 - ✓ $I(t) = C \frac{dv}{dt} + \frac{1}{L} \int Vdt + \frac{1}{R} V$
- Draw the equivalent electrical analogous circuit. (6 Marks)
 - ✓ Force voltage electrical circuits:



- ✓ Force current electrical circuits:



3. Write differential equations governing the mechanical rotational system shown in Fig. below. Draw the electrical equivalent analog circuits (current and voltage) (Nov/Dec-16)



- To find equation on J_1 and J_2 . (5 Marks)
 - ✓ $J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1[\theta_1 - \theta_2] = T$
 - ✓ $J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_1[\theta_2 - \theta_1] + K_2\theta_2 = 0$
- Force voltage and force Current equation. (5 Marks)

(i) Force voltage equation:

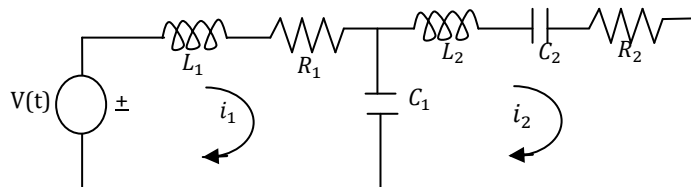
- ✓ $V(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int (i_1 - i_2) dt$
- ✓ $L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_1} \int (i_2 - i_1) dt + \frac{1}{C_2} \int i_2 dt = 0$

(ii) Force current equation:

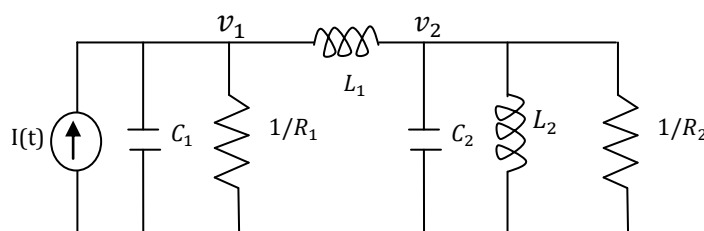
- ✓ $I(t) = C_1 \frac{dv_1}{dt} + G_1 v_1 + \frac{1}{L_1} \int (v_1 - v_2) dt$
- ✓ $C_2 \frac{dv_2}{dt} + G_2 v_2 + \frac{1}{L_1} \int (v_2 - v_1) dt + \frac{1}{L_2} \int v_2 dt = 0$

- Draw the equivalent electrical analogous circuit. (6 Marks)

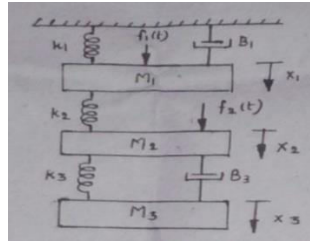
- ✓ Force voltage electrical circuits:



- ✓ Force current electrical circuits:



4. Write the differential equations governing the mechanical translational system as shown in figure. Draw the Force-Voltage and Force-Current electrical analogous circuits and verify by mesh and node equations. (Nov/Dec-15)



- To find equation on M_1, M_2 and M_3 . (5 Marks)

✓ $M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1x_1 + K_2(x_1 - x_2) = f_1(t)$

✓ $M_2 \frac{d^2x_2}{dt^2} + K_2(x_2 - x_1) + K_3(x_2 - x_3) + B_3 \frac{d(x_2 - x_3)}{dt} = f_2(t)$

✓ $M_3 \frac{d^2x_3}{dt^2} + B_3 \frac{d(x_3 - x_2)}{dt} + K_3(x_3 - x_2) = 0$

- Force voltage and force Current equation. (5 Marks)

(i) Force voltage equation:

✓ $V_1(t) = L_1 \frac{di_1}{dt} + \frac{1}{C_1} \int i_1 dt + R_1 i_1 + \frac{1}{C_2} \int (i_1 - i_2) dt$

✓ $V_2(t) = L_2 \frac{di_2}{dt} + \frac{1}{C_2} \int (i_2 - i_1) dt + R_3 (i_2 - i_3) + \frac{1}{C_3} \int (i_2 - i_3) dt$

✓ $L_3 \frac{di_3}{dt} + \frac{1}{C_3} \int (i_3 - i_2) dt + R_3 (i_3 - i_2) = 0$

(ii) Force current equation:

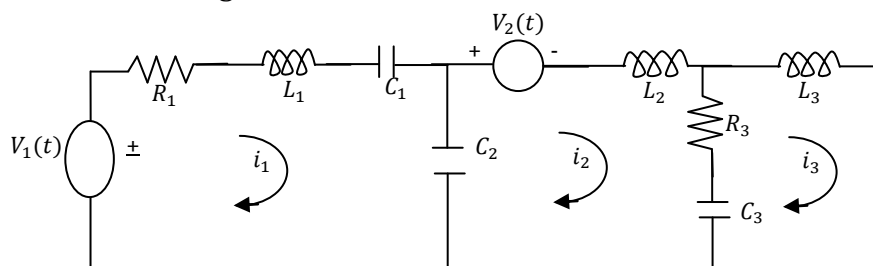
✓ $I_1(t) = C_1 \frac{dv_1}{dt} + \frac{1}{L_1} \int v_1 dt + G_1 v_1 + \frac{1}{L_2} \int (v_1 - v_2) dt$

✓ $I_2(t) = C_2 \frac{dv_2}{dt} + \frac{1}{L_2} \int (v_2 - v_1) dt + G_3 (v_2 - v_3) + \frac{1}{L_3} \int (v_2 - v_3) dt$

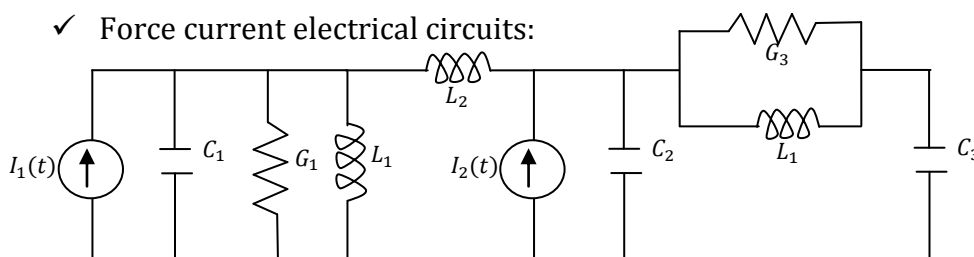
✓ $C_3 \frac{dv_3}{dt} + \frac{1}{L_3} \int (v_3 - v_2) dt + G_3 (v_3 - v_2) = 0$

- Draw the equivalent electrical analogous circuit. (6 Marks)

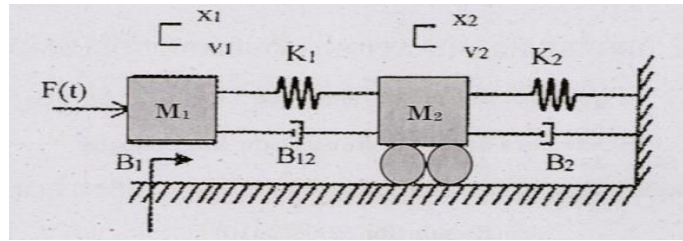
✓ Force voltage electrical circuits:



✓ Force current electrical circuits:



5. Write the differential equations governing the mechanical translational system as shown in figure. Draw the Force-Voltage and Force-Current electrical analogous circuits and verify by mesh and node equations. (Nov/Dec-14)



- To find equation on M_1, M_2 and M_3 . (5 Marks)

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d(x_1-x_2)}{dt} + K_1(x_1 - x_2) = F(t)$$

$$M_2 \frac{d^2x_2}{dt^2} + K_2x_2 + B_2 \frac{dx_2}{dt} + B_{12} \frac{d(x_2-x_1)}{dt} + K_1(x_2 - x_1) = 0$$

- Force voltage and force Current equation. (5 Marks)

(i) Force voltage equation:

$$V(t) = L_1 \frac{di_1}{dt} + R_1i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1} \int (i_1 - i_2) dt$$

$$L_2 \frac{di_2}{dt} + \frac{1}{C_1} \int (i_2 - i_1) dt + R_2i_2 + R_{12}(i_2 - i_1) + \frac{1}{C_2} \int i_2 dt = 0$$

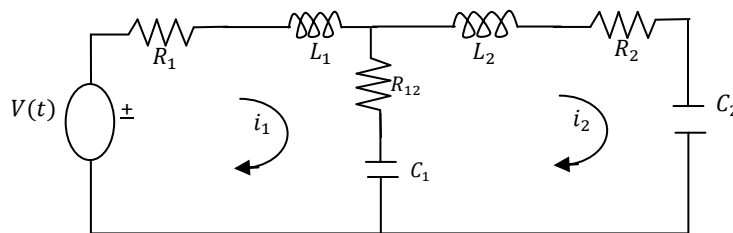
(ii) Force current equation:

$$I(t) = C_1 \frac{dv_1}{dt} + G_1v_1 + G_{12}(v_1 - v_2) + \frac{1}{L_1} \int (v_1 - v_2) dt$$

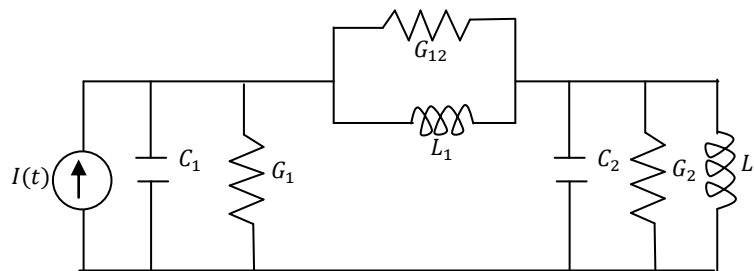
$$C_2 \frac{dv_2}{dt} + \frac{1}{L_1} \int (v_2 - v_1) dt + G_2v_2 + G_{12}(v_2 - v_1) + \frac{1}{L_2} \int v_2 dt = 0$$

- Draw the equivalent electrical analogous circuit. (6 Marks)

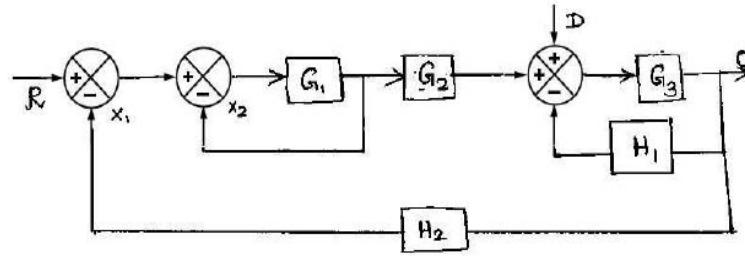
- Force voltage electrical circuits:



- Force current electrical circuits:

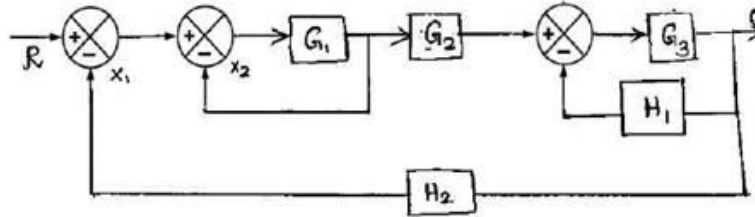


6. For the block diagram shown below, find the output C due to R and disturbance D. (May/Jun-11, Nov/Dec-12)



• Case(i): (8 Marks)

✓ When R is acting alone; D=0, therefor the block diagram is reduced as below

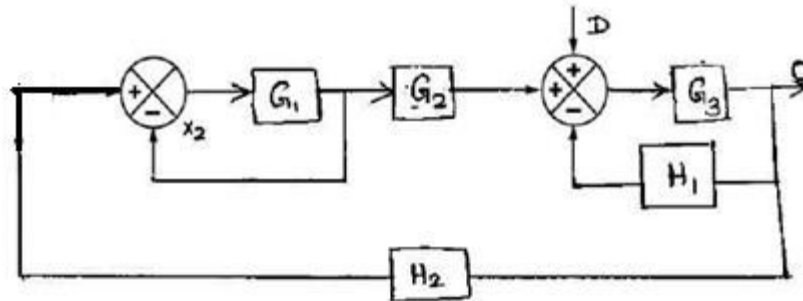


✓ Use different block reduction techniques and find the transfer function

$$\frac{C}{R} = \frac{G_1 G_2 G_3}{1 + G_1 + G_3 H_1 - G_1 G_3 H_1 + G_1 G_2 G_3 H_2}$$

• Case(ii): (8 Marks)

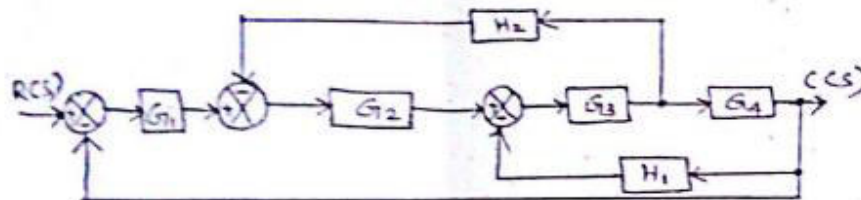
✓ When D is acting alone; R=0, therefor the block diagram is reduced as below



✓ Use different block reduction techniques and find the transfer function

$$\frac{C}{D} = \frac{G_3 (1 + G_1)}{1 + G_1 + G_3 H_1 + G_1 G_3 H_1 + G_1 G_2 G_3 H_2}$$

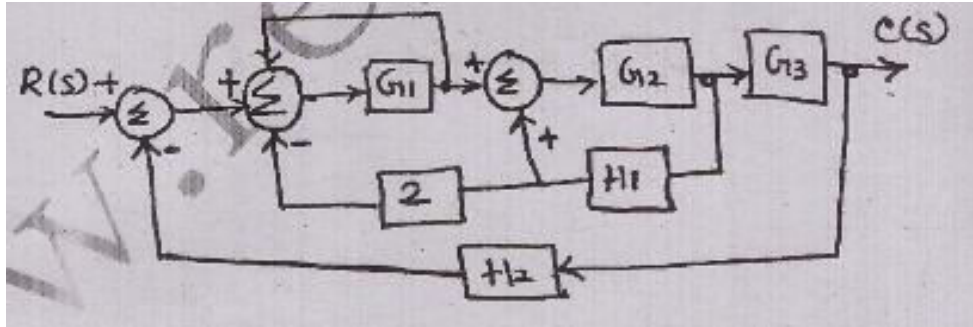
7. Reduce the block diagram shown below and find C/R. (Nov/Dec-16)



✓ Use different block reduction techniques and find the transfer function

$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3 G_4}{1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4}$$

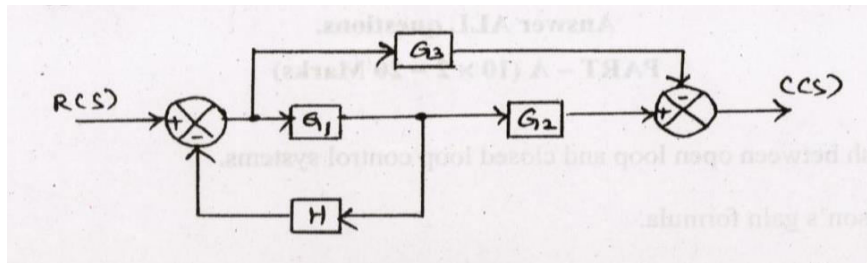
8. Find the transfer function of the system shown in fig. using block diagram reduction technique and signal flow graph technique. (May/Jun-15)



✓ Use different block reduction techniques and find the transfer function

$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3}{(1 - G_1)(1 - G_2 H_1) + 2G_1 G_2 H_1 + G_1 G_2 G_3 H_2}$$

9. Convert the block diagram shown in figure to signal flow graph and find the transfer function using Mason's gain formula. Verify with Block diagram approach. (May/Jun-16)



• Block diagram reduction method: (8 Marks)

✓ Solve the problem using block reduction techniques and find the transfer function

$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 + G_3}{1 - G_1 H}$$

• Signal flow graph: (8 Marks)

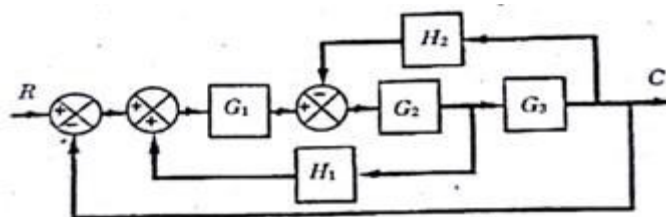
✓ Solve the problem using signal flow graph techniques and find the transfer function

Formula: (2 Marks)

$$T = \frac{1}{\Delta} P_1 \Delta_1$$

$$T = \frac{G_1 G_2 + G_3}{1 - G_1 H}$$

10. Simplify the following diagram using block diagram reduction method. Also derive the transfer function of the same using signal flow graph. (Apr/May-17)



• Block diagram reduction method: (8 Marks)

✓ Solve the problem using block reduction techniques and find the transfer function

$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - G_1 G_2 H_1 + G_1 G_2 G_3}$$

• **Signal flow graph: (8 Marks)**

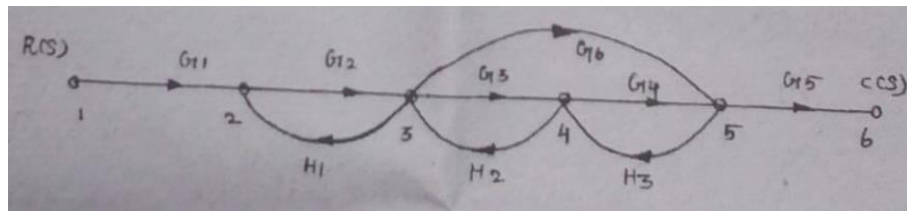
✓ Solve the problem using signal flow graph techniques and find the transfer function

Formula: (2 Marks)

$$T = \frac{1}{\Delta} P_1 \Delta_1$$

$$T = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - G_1 G_2 H_1 + G_1 G_2 G_3}$$

11. The signal flow graph for a feedback system is shown in fig. Determine the closed loop transfer function C(S)/R(S). (Nov/Dec-15)



• **Signal flow graph:**

✓ Find forward path gain: (3 Marks)

✓ Find individual loop gain: (3 Marks)

✓ Find product of two non touching loops: (3 Marks)

✓ Calculate Δ and Δ_k : (3 Marks)

✓ Solve the problem using signal flow graph techniques and find the transfer function

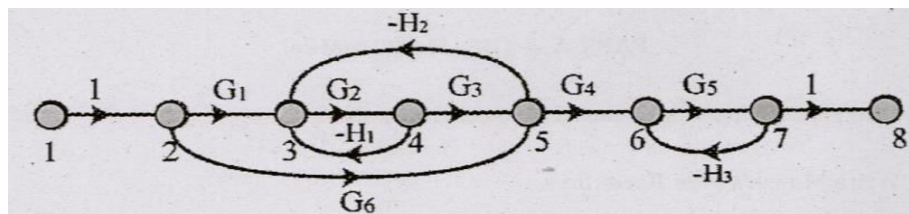
Formula: (3 Marks)

$$T = \frac{1}{\Delta} P_1 \Delta_1$$

✓ Transfer function: (1 Mark)

$$T = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_2 G_5 G_6}{1 - G_2 H_1 - G_3 H_2 - G_4 H_3 - G_6 H_2 H_3 + G_2 G_4 H_1 H_3}$$

12. The signal flow graph for a feedback control system is shown in figure. Determine the closed loop transfer function C(s)/R(s). (Nov/Dec-14)



• **Signal flow graph:**

✓ Find forward path gain: (3 Marks)

✓ Find individual loop gain: (3 Marks)

✓ Find product of two non touching loops: (3 Marks)

✓ Calculate Δ and Δ_k : (3 Marks)

- ✓ Solve the problem using signal flow graph techniques and find the transfer function

Formula: (3 Marks)

$$T = \frac{1}{\Delta} P_1 \Delta_1$$

- ✓ Transfer function: (1 Mark)

$$T = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 - G_1 G_2 H_1 + G_1 G_2 G_3}$$

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EC6405 - CONTROL SYSTEM ENGINEERING

QUESTION BANK

UNIT - II TIME RESPONSE ANALYSIS

PART - A

1. What is transient response & steady state response? (Nov/Dec-12)

The transient response is the response of the system when the system changes from one state to another.

The steady state response is the response of the system when it approaches infinity.

2. What is an order of a system? (Nov/Dec-15)

The order of a system is the order of the differential equation governing the system. The order of the system can be obtained from the transfer function of the given system.

3. Define Damping ratio.

Damping ratio is defined as the ratio of actual damping to critical damping.

4. List the time domain specifications. (Nov/Dec-16, Nov/Dec-15, Nov/Dec-14)

The time domain specifications are

- i. Rise Time
- ii. Peak time
- iii. Delay Time
- iv. Peak overshoot

5. Define Delay time, Rise time, Peak time, Peak overshoot, Settling time. (May/Jun-14, Nov/Dec-12, Nov/Dec-11, Nov/Dec-10)

Delay time: The time taken for response to reach 50% of final value for the very first time is delay time.

Rise time: The time taken for response to raise from 0% to 100% for the very first time is rise time.

Peak time: The time taken for the response to reach the peak value for the first time is peak time.

Peak overshoot: Peak overshoot is defined as the ratio of maximum peak value measured from the maximum value to final value.

Settling time: Settling time is defined as the time taken by the response to reach and stay within specified error.

6. Define Steady state error. (Apr/May-17, Nov/Dec-16, Nov/Dec-15, Nov/Dec-10)

The steady state error is defined as the value of error as time tends to infinity.

7. What are static error constants? (May/Jun-13, Nov/Dec-13)Positional error constant - K_p Velocity error constant - K_v Acceleration error constant K_a **8. What are the different types of controllers?**

Proportional controller

PI controller

PD controller

PID controller

9. Why derivative controller is not used in control systems? (May/Jun-12, May/Jun-11)

The derivative controller produces a control action based on the rate of change of error signal and it does not produce corrective measures for any constant error.

10. Name the test signals used in control system. (Nov/Dec-15, May/Jun-16)

The commonly used test input signals in control system are impulse step ramp acceleration and sinusoidal signals.

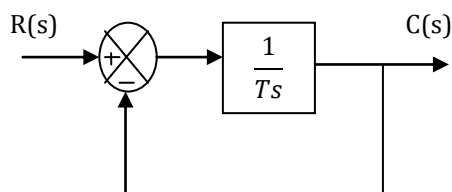
11. What is the effect of PD, PI controller on system performance? (Nov/Dec-16)

The effect of PD controller is to increase the damping ratio of the system and so the peak overshoot is reduced.

The PI controller increases the order of the system by one, which results in reducing the steady state error. But the system becomes less stable than the original system.

PART - B (Answers as Hint)**1. (a) Derive the expressions and draw the response of first order system for unit step input. (May/Jun-16, Nov/Dec-15)**

- Draw the block diagram: (2 Marks)



- Find closed loop transfer function: (2 Marks)

$$\checkmark \frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

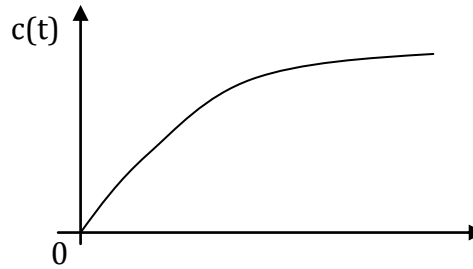
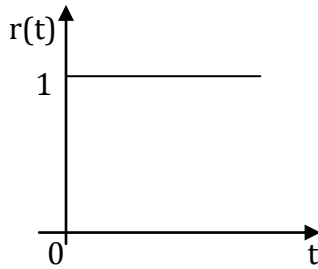
$$\checkmark \text{Let } R(S) = \frac{1}{s}$$

$$\checkmark C(S) = \frac{1/T}{s(s+\frac{1}{T})}$$

- Find the response in time domain: (2 Marks)

$$\checkmark c(t) = 1 - e^{-\frac{t}{T}}$$

- Draw the response of first order system: (2 Marks)



(b) The unity feedback system is characterized by an open loop transfer function $G(S) = \frac{K}{S(S+10)}$. Determine the gain K, so that the system will have a damping ratio of 0.5. For this value of K, determine settling time, peak overshoot and time to peak overshoot for a unit step input. (Apr/May-17, May/Jun-16, May/Jun-14, Nov/Dec-12, May/Jun-11)

- Given: (2 Marks)

✓ $G(S) = \frac{K}{S(S+10)}$, $H(S)=1$

✓ $\zeta = 0.5$

- Solution: (6 Marks)

- ✓ To find closed loop transfer function: (2 Mark)

$$\frac{C(S)}{R(S)} = \frac{K}{S^2 + 10S + K}$$

- ✓ To find K: (1 Mark)

$$\omega_n = \sqrt{K}, K=100$$

- ✓ To find percentage peak overshoot: (1 Marks)

$$\%M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = 16.3\%$$

- ✓ To find time to peak overshoot: (1 Marks)

$$t_p = \frac{\pi}{\omega_d} = 0.363 \text{ sec.}$$

- ✓ To find settling time: (1 Marks)

$$t_s = \frac{3}{\zeta\omega_n} \text{ for } 5\% = 0.6 \text{ sec}$$

$$= \frac{4}{\zeta\omega_n} \text{ for } 2\% = 0.8 \text{ sec}$$

2. Draw the response of second order system for all case and when input is unit step. (Nov/Dec-16, May/Jun-15, May/Jun-13)

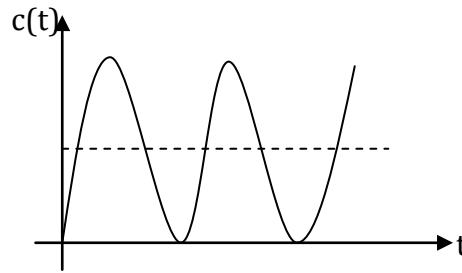
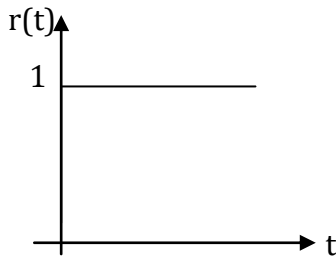
Consider Standard equation: $\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$, $r(t) = 1$, $R(S) = \frac{1}{S}$

- Case (i): Un damped system: $\zeta = 0$ (4 Marks)

✓ Equation: $\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + \omega_n^2}$

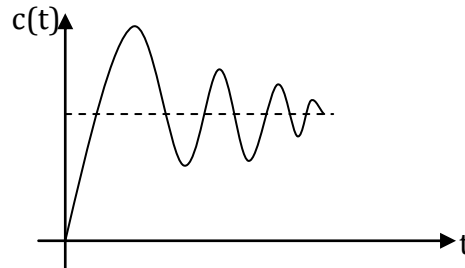
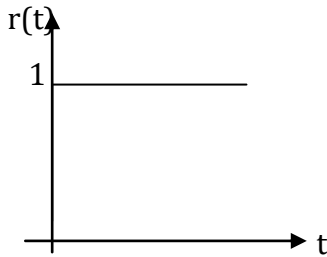
✓ Find the response in S domain: $C(S) = \frac{1}{S} \frac{\omega_n^2}{S^2 + \omega_n^2}$

- ✓ Find the response in time domain: $c(t) = 1 - \cos\omega_n t$
- ✓ Draw the response of first order system



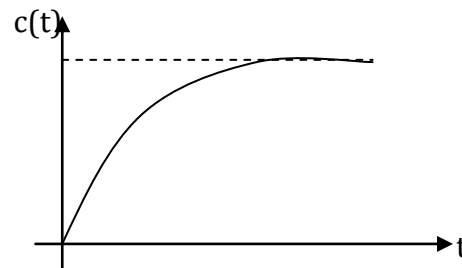
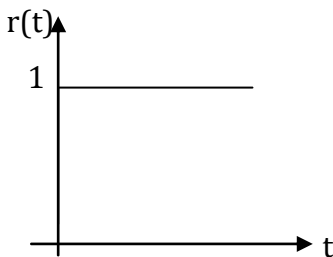
• **Case (ii): Under damped system: $0 < \zeta < 1$ (4 Marks)**

- ✓ Equation: $\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
- ✓ Find the response in S domain: $C(S) = \frac{1}{s} \left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right)$
- ✓ Find the response in time domain: $c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$
- ✓ Draw the response of first order system



• **Case (iii): Critically damped system: $\zeta = 1$ (4 Marks)**

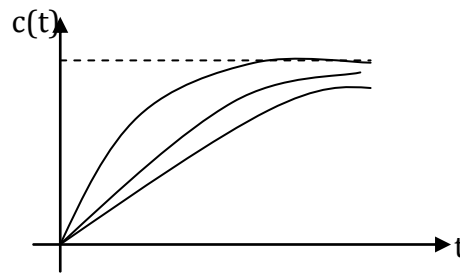
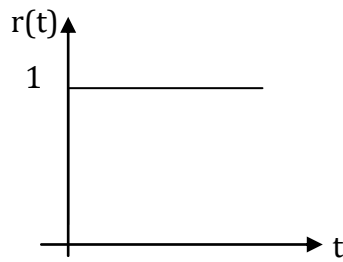
- ✓ Equation: $\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
- $\frac{C(S)}{R(S)} = \frac{\omega_n^2}{(s+s_1)(s+s_2)}$ Where, $s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$, $s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$
- ✓ Find the response in S domain: $C(S) = \frac{\omega_n^2}{s(s+s_1)(s+s_2)}$
- ✓ Find the response in time domain: $c(t) = 1 - \frac{\omega_n}{2\sqrt{\zeta^2 - 1}} \left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2} \right)$
- ✓ Draw the response of first order system



• **Case (iv): Over damped system: $\zeta > 1$ (4 Marks)**

- ✓ Equation: $\frac{C(S)}{R(S)} = \frac{\omega_n^2}{(s+\omega_n)^2}$
- ✓ Find the response in S domain: $C(S) = \frac{1}{s} \frac{\omega_n^2}{(s+\omega_n)^2}$
- ✓ Find the response in time domain: $c(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)$

- ✓ Draw the response of first order system



3. For a unity feedback control system the open loop transfer function $G(s) = \frac{10(s+2)}{s^2(s+1)}$. Find

(a) The position, velocity and acceleration error constants.

(b) The steady state error when the input is $R(S)$ where $R(S) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$ (Nov/Dec-16)

- Given: (2 Marks)

- ✓ $G(S) = \frac{10(s+2)}{s^2(s+1)}$,

- ✓ $H(S)=1$

- ✓ $R(S) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$

- Solution: (14 Marks)

- ✓ To find position error constants: (3 Mark)

$$K_p = \lim_{s \rightarrow 0} G(S)H(S) = \infty.$$

- ✓ To find velocity error constants: (3 Mark)

$$K_v = \lim_{s \rightarrow 0} sG(S)H(S) = \infty.$$

- ✓ To find acceleration error constants: (3 Marks)

$$K_a = \lim_{s \rightarrow 0} s^2G(S)H(S) = 20.$$

- ✓ To find error signal in S-domain: (3 Marks)

$$E(S) = \frac{R(S)}{1+G(S)H(S)} = \frac{\left(\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}\right)}{\left(\frac{s^2(s+1)+10(s+2)}{s^2(s+1)}\right)}$$

- ✓ To the steady state error:(2 Marks)

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \frac{1}{60}$$

4. A unity feedback system has the forward transfer function $G(S) = \frac{KS}{(1+S)^2}$. For the input $r(t) = 1 + 5t$, find the minimum value of K so that the steady state error is less than 0.1 (use final value theorem). (May/Jun-15)

- Given: (4 Marks)

- ✓ $G(S) = \frac{KS}{(1+S)^2}$.

- ✓ $H(S)=1$

- ✓ $r(t) = 1 + 5t$

- ✓ $e_{ss} = 0.1$

- **Solution: (12 Marks)**

- ✓ **To find R(S) (4 Mark)**

$$R(S) = \frac{1}{S} + \frac{5}{S^2}$$

- ✓ **To find error signal in S-domain: (4 Marks)**

$$\begin{aligned} E(S) &= \frac{R(S)}{1+G(S)H(S)} \\ &= \frac{\frac{1}{S} + \frac{5}{S^2}}{1 + \frac{KS}{(1+S)^2}} \end{aligned}$$

- ✓ **To the steady state error:(4 Marks)**

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \infty$$

5. Determine the steady state errors for the following inputs $5u(t)$, $5tu(t)$, $5t^2u(t)$ to a system whose open loop transfer function is given by $G(S) = \frac{[100(S+2)(S+6)]}{[S(S+3)(S+4)]}$ (Nov/Dec-14)

- **Given: (1 Marks)**

- ✓ $G(S) = \frac{[100(S+2)(S+6)]}{[S(S+3)(S+4)]}$,

- ✓ $H(S)=1$

- **Solution: (15 Marks)**

- ✓ **Case (i): Input is $5u(t)$: (5 Marks)**

$$R(S) = \frac{5}{S}$$

To find error signal in S-domain:

$$\begin{aligned} E(S) &= \frac{R(S)}{1+G(S)H(S)} \\ &= \frac{\frac{5}{S}}{1 + \frac{[100(S+2)(S+6)]}{[S(S+3)(S+4)]}} \end{aligned}$$

To the steady state error:

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = 0$$

- ✓ **Case (ii): Input is $5tu(t)$: (5 Marks)**

$$R(S) = \frac{5}{S^2}$$

To find error signal in S-domain:

$$\begin{aligned} E(S) &= \frac{R(S)}{1+G(S)H(S)} \\ &= \frac{\frac{5}{S^2}}{1 + \frac{[100(S+2)(S+6)]}{[S(S+3)(S+4)]}} \end{aligned}$$

To the steady state error:

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \frac{1}{20}$$

✓ **Case (iii): Input is $5t^2u(t)$: (5 Marks)**

$$R(S) = \frac{10}{S^3}$$

To find error signal in S-domain:

$$\begin{aligned} E(S) &= \frac{R(S)}{1+G(S)H(S)} \\ &= \frac{\frac{10}{S^3}}{1+\frac{[100(S+2)(S+6)]}{[S(S+3)(S+4)]}} \end{aligned}$$

To the steady state error:

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \infty$$

6. The open loop transfer function of a unity feedback system is given by $G(S) = \frac{20}{S(S+2)}$. The input function is $r(t) = 2 + 3t + t^2$. Determine generalized error coefficient and steady state error. (May/Jun-14)

• **Given: (2 Marks)**

✓ $G(S) = \frac{20}{S(S+2)}$,

✓ $H(S)=1$

✓ $r(t) = 2 + 3t + t^2$

• **Solution: (14 Marks)**

✓ **To find steady state errors: (7 Marks)**

Input is $r(t) = 2 + 3t + t^2$

$$R(S) = \frac{2}{S} + \frac{3}{S^2} + \frac{2}{S^3}$$

To find error signal in S-domain:

$$\begin{aligned} E(S) &= \frac{R(S)}{1+G(S)H(S)} \\ &= \frac{\frac{2}{S} + \frac{3}{S^2} + \frac{2}{S^3}}{1 + \frac{20}{S(S+2)}} \end{aligned}$$

To the steady state error:

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \infty$$

✓ **To find generalized error coefficient: (7 Marks)**

To find $F(S) = \frac{1}{1+G(S)H(S)}$

$$= \frac{1}{1 + \frac{20}{S(S+2)}} = \frac{S(S+2)}{S(S+2)+20}$$

To find $C_0 = \lim_{s \rightarrow 0} F(S) = 0$

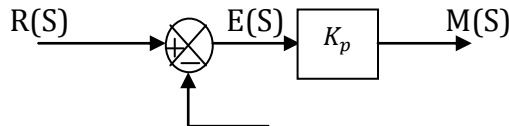
To find $C_1 = \lim_{s \rightarrow 0} \frac{d}{ds} F(S) = \frac{1}{10}$

To find $C_2 = \lim_{s \rightarrow 0} \frac{d^2}{ds^2} F(S) = \frac{9}{100}$

7. With suitable block diagrams and equations, explain the following types of controllers employed in control systems: a) Proportional controller, b) Proportional-plus-integral controller, c) PID controller, d) Integral controller (May/Jun-16, Nov/Dec-15, Nov/Dec-12, May/Jun-11, Nov/Dec-10)

a) Proportional controller: (4 Marks)

- ✓ Output is proportional to error signal input.
- ✓ Block diagram:



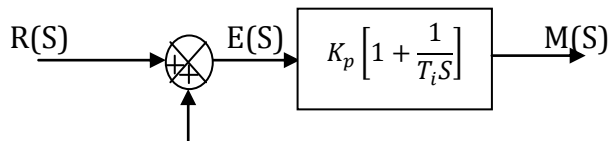
- ✓ Equation:

$$M(S) = K_p E(S)$$

$$K_p = \frac{M(S)}{E(S)}$$

b) Proportional-plus-Integral controller: (4 Marks)

- ✓ Output is proportional to error signal input and integral component.
- ✓ Block diagram:



- ✓ Equation:

$$M(t) = K_p \int e(t) dt + K_p e(t)$$

$$M(S) = K_p \frac{E(S)}{T_i S} + K_p E(S)$$

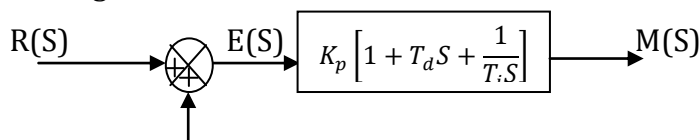
$$\frac{M(S)}{E(S)} = K_p \left[1 + \frac{1}{T_i S} \right]$$

- ✓ Effects:

The PI controller increases the order of the system by one, which results in reducing the steady state error. But the system becomes less stable than the original system.

c) PID controller: (4 Marks)

- ✓ It is combination of proportional control action, derivative control action and integral control action.
- ✓ Block diagram:



✓ Equation:

$$M(t) = K_p e(t) + K_i \int e(t) dt + K_d E(t)$$

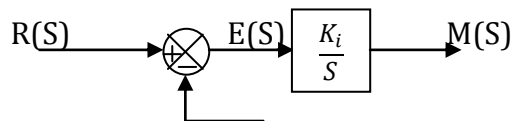
$$M(S) = K_p E(S) + K_p \frac{E(S)}{T_i S} + K_p T_d S E(S)$$

$$\frac{M(S)}{E(S)} = K_p \left[1 + T_d S + \frac{1}{T_i S} \right]$$

d) Integral controller: (4 Marks)

✓ Output is proportional to integral of error signal input.

✓ Block diagram:



✓ Equation:

$$M(t) = K_i \int e(t) dt$$

$$\frac{M(S)}{E(S)} = \frac{K_i}{S}$$

8. Explain about briefly the operation of P, PI, and PID control compensation using MATLAB programs. (Apr/May-17)

• **MATLAB programs P control:**

`Kp = 10;`

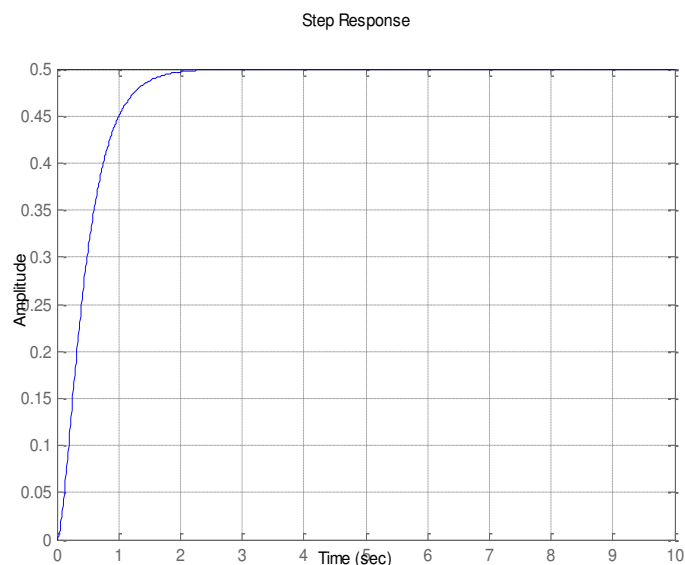
`G = tf(Kp,1);`

`t = 0:0.01:10;`

`L = feedback (G*H,1)`

`figure; step(L,t)`

`grid`



- **MATLAB programs PI control:**

```
Kp = 100;
```

```
Ki = 10;
```

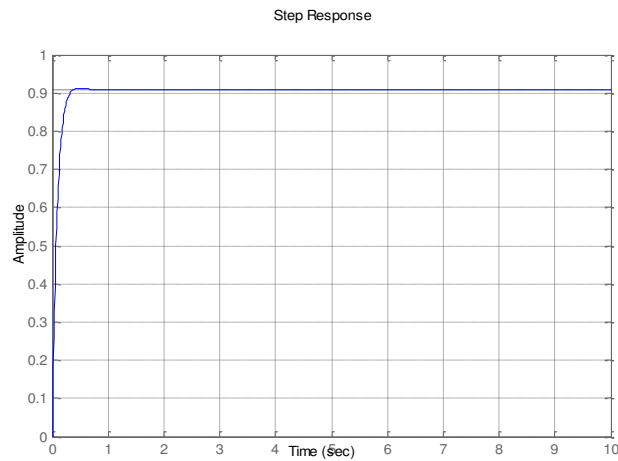
```
G = tf([Ki Kp],1)
```

```
t = 0:0.01:10;
```

```
L = feedback(G*H,1)
```

```
figure; step(L,t)
```

```
grid
```



- **MATLAB programs PID control:**

```
Kp = 100;
```

```
Kd = 10;
```

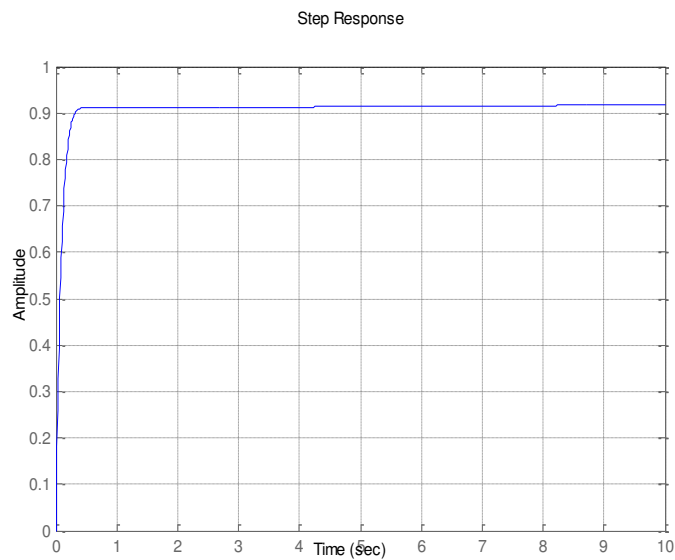
```
Ki = 1;
```

```
G = tf([Kd Kp Ki],[1 0])
```

```
t = 0:0.01:10;
```

```
L = feedback(G*H,1)
```

```
figure; step(L,t)
```



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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EC6405 - CONTROL SYSTEM ENGINEERING

QUESTION BANK

UNIT - III FREQUENCY RESPONSE ANALYSIS

PART -A

1. What is frequency response and List out the different frequency domain specifications?

(May/Jun-16, Nov/Dec-11, Nov/Dec-10)

A frequency response is the steady state response of a system when the input to the system is a sinusoidal signal.

The frequency domain specifications are resonant peak, resonant frequency, Bandwidth, cutoff rate, Gain margin and Phase margin.

2. Define Resonant Peak (Δ_r), resonant frequency (Δf).

The maximum value of the magnitude of closed loop transfer function is called Resonant Peak.

The frequency at which resonant peak occurs is called resonant frequency.

3. What is Bandwidth?

The Bandwidth is the range of frequencies for which the system gain is more than 3 dB. The bandwidth is a measure of the ability of a feedback system to reproduce the input signal noise rejection characteristics and rise time.

4. Define Gain Margin. (Apr/May-17, May/Jun-14, Nov/Dec-14, Nov/Dec-13)

The Gain Margin, k_g is defined as the reciprocal of the magnitude of the open loop transfer function at phase cross over frequency.

Gain margin $k_g = 1 / \Delta G (j\Delta\omega_c)$.

5. What is Phase margin? (Apr/May-17, May/Jun-14, Nov/Dec-14, Nov/Dec-13)

The Phase margin is the amount of phase lag at the gain cross over frequency required to bring system to the verge of instability.

6. What is Bode plot?

The Bode plot is the frequency response plot of the transfer function of a system. A Bode plot consists of two graphs. One is the plot of magnitude of sinusoidal transfer function versus $\log \Delta$. The other is a plot of the phase angle of a sinusoidal function versus $\log \Delta$.

7. Define Corner frequency. (May/Jun-14, May/Jun-11, Nov/Dec-12)

The frequency at which the two asymptotic meet in a magnitude plot is called Corner frequency.

8. What are M and N circles? (May/Jun-16, Nov/Dec-15, Nov/Dec-13)

The magnitude of closed loop transfer function with unit feedback can be shown for every value of M. These circles are called M circles.

If the phase of closed loop transfer function with unity feedback is α , then it can be shown that $\tan \alpha$ will be in the form of circle for every value of α . These circles are called N circles

9. What are Nichols charts and its advantage or uses? (May/Jun-15, Nov/Dec-14, Nov/Dec-12, May/Jun-12, Nov/Dec-10)

The chart consisting if M & N loci in the log magnitude versus phase diagram is called Nichols chart.

The advantages are it is used to find the closed loop frequency response from open loop frequency response.

10. What is a compensator? List the types of compensators?

A device inserted into the system for the purpose of satisfying the specifications is called as a compensator.

- i. Lag compensator
- ii. Lead compensator
- iii. Lag-Lead compensator.

PART - B(Answers as Hint)

1. Sketch the bode plot for the following transfer function and determine the phase margin & gain margin. $G(S) = \frac{20}{S(1+3S)(1+4S)}$ (Nov/Dec-15, Nov/Dec-13)

- **Given: (2 Marks)**

$$G(S) = \frac{20}{S(1+3S)(1+4S)}$$

- **Solution: (14 Marks)**

- ✓ **Step: 1 (1 Mark)**

Sub $S=j\omega$

$$G(j\omega) = \frac{20}{j\omega(1+3j\omega)(1+4j\omega)}$$

- ✓ **Step: 2 (2 Mark)**

Find the corner frequency, slope in db:

$$\omega_{c1} = \frac{1}{4} = 0.25, \omega_{c2} = \frac{1}{3} = 0.33$$

Choose $\omega_l = 0.15 \text{ rad/sec}$, $\omega_h = 1 \text{ rad/sec}$

Term	Corner frequency	Slope in db	Change in slope	Phase
$\frac{20}{j\omega}$	-	-20	-	-90
$\frac{1}{1+j4\omega}$	0.25	-20	-20-20=-40	$-\tan^{-1} 4\omega$
$\frac{1}{1+j3\omega}$	0.33	-20	-40-20=-60	$-\tan^{-1} 3\omega$

✓ **Step: 3 (3 Mark)**

Find the gain:

$$\text{At } \omega = \omega_l \text{ } A = |G(j\omega)| = 20 \log \left| \frac{20}{0.15} \right| = 42.5 \text{ db}$$

$$\text{At } \omega = \omega_{c1} \text{ } A = |G(j\omega)| = 20 \log \left| \frac{20}{0.25} \right| = 38 \text{ db}$$

$$\text{At } \omega = \omega_{c2} \text{ } A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A_{(\text{At } \omega = \omega_{c1})} = 33 \text{ db}$$

$$\text{At } \omega = \omega_h \text{ } A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{(\text{At } \omega = \omega_{c2})} = 4 \text{ db}$$

✓ **Step: 4 (3 Mark)**

Find phase angle:

$$\varphi = -90^\circ - \tan^{-1} 3\omega - \tan^{-1} 4\omega$$

ω	φ
0.15	-145.18 \approx -146
0.2	-159.61 \approx -160
0.25	-171.86 \approx -172
0.33	-187.5 \approx -188
0.6	-218.32 \approx -218
1	-237.56 \approx -238

✓ **Step: 5 (5 Mark)**

Draw the bode plot:

ω	Gain in db	φ
0.15	42.5	-146
0.25	38	-172
0.33	33	-188
1	4	-238

Find gain margin & phase Margin:

$$\text{Gain cross over frequency: } \omega_{gc} = 1.1 \text{ rad/sec}$$

$$\text{Phase cross over frequency: } \varphi_{gc} = 0.29 \text{ rad/sec}$$

$$\text{Gain Margin: } K_g = 35 \text{ db}$$

$$\text{Phase Margin: } \gamma = \infty$$

2. Sketch Bode plot for the following transfer function and determine the system gain K for the

gain cross over frequency to be 5 rad/sec $G(S) = \frac{KS^2}{(1+0.2S)(1+0.02S)}$. (May/Jun-16, Nov/Dec-10)

- Given: (2 Marks)

$$G(S) = \frac{KS^2}{(1+0.2S)(1+0.02S)}$$

- Solution: (14 Marks)

- ✓ Step: 1 (1 Mark)

Let K=1

$$\text{Sub } S=j\omega \quad G(j\omega) = \frac{(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

- ✓ Step: 2 (2 Mark)

Find the corner frequency, slope in db:

$$\omega_{c1} = \frac{1}{0.2} = 5, \quad \omega_{c2} = \frac{1}{0.02} = 50$$

Choose $\omega_l = 0.1 \text{ rad/sec}$, $\omega_h = 100 \text{ rad/sec}$

Term	Corner frequency	Slope in db	Change in slope	Phase
ω^2	-	+40	-	180
$\frac{1}{1+j0.2\omega}$	5	-20	40-20=20	$-\tan^{-1} 0.2\omega$
$\frac{1}{1+j0.02\omega}$	50	-20	20-20=0	$-\tan^{-1} 0.02\omega$

- ✓ Step: 3 (3 Mark)

Find the gain:

$$\text{At } \omega = \omega_l \quad A = |G(j\omega)| = 20 \log|0.1^2| = -40 \text{ db}$$

$$\text{At } \omega = \omega_{c1} \quad A = |G(j\omega)| = 20 \log|5^2| = 28 \text{ db}$$

$$\text{At } \omega = \omega_{c2} \quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A_{(\text{At } \omega = \omega_{c1})} = 48 \text{ db}$$

$$\text{At } \omega = \omega_h \quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{(\text{At } \omega = \omega_{c2})} = 48 \text{ db}$$

- ✓ Step: 4 (3 Mark)

Find phase angle:

$$\varphi = 180^\circ - \tan^{-1} 0.2\omega - \tan^{-1} 0.02\omega$$

ω	φ
0.1	178
1	168
5	130
10	106
50	50
100	30

✓ **Step: 5 (5 Mark)**

Draw the bode plot:

ω	Gain in db	ϕ
0.1	-40	178
5	28	130
50	48	50
100	48	30

Find K if gain cross over frequency = 5 rad/sec:

At $\omega = 5 \text{ rad/sec}$ Gain = 28db

$$20 \log K = 28$$

$$K = 10^{-\left(\frac{28}{20}\right)} = 0.0398$$

3. Given $G(S) = \frac{Ke^{-0.2s}}{s(s+2)(s+8)}$ find K for the following two cases:

- i. Gain margin equal to 6 db,
- ii. Phase margin equal to 45°. (Nov/Dec-12, May/Jun-11)

• **Given: (2 Marks)**

$$G(S) = \frac{Ke^{-0.2s}}{s(s+2)(s+8)}$$

• **Solution: (14 Marks)**

✓ **Step: 1 (1 Mark)**

Let K=1 and

$$G(S) = \frac{0.0625e^{-0.2s}}{s(1+0.5s)(1+0.125s)}$$

Sub S=j ω

$$G(j\omega) = \frac{0.0625e^{-0.2j\omega}}{j\omega(1+0.5j\omega)(1+0.125j\omega)}$$

✓ **Step: 2 (2 Mark)**

Find the corner frequency, slope in db:

$$\omega_{c1} = \frac{1}{0.5} = 2, \omega_{c2} = \frac{1}{0.125} = 8, \text{ Choose } \omega_l = 0.5 \text{ rad/sec, } \omega_h = 50 \text{ rad/sec}$$

Term	Corner frequency	Slope in db	Change in slope	Phase
$e^{-0.2j\omega}$	-	-	-	$-0.2\omega X \frac{180}{\pi}$
$\frac{0.0625}{j\omega}$	-	-20	-20	-90°
$\frac{1}{1+j0.5\omega}$	2	-20	$-20-20=-40$	$-\tan^{-1} 0.5\omega$
$\frac{1}{1+j0.125\omega}$	8	-20	$-40-20=-60$	$-\tan^{-1} 0.125\omega$

✓ **Step: 3 (3 Mark)**

Find the gain:

$$\text{At } \omega = \omega_l \text{ } A = |G(j\omega)| = 20 \log \left| \frac{0.0625}{0.5} \right| = -18 \text{ db}$$

$$\text{At } \omega = \omega_{c1} \text{ } A = |G(j\omega)| = 20 \log \left| \frac{0.0625}{2} \right| = -30 \text{ db}$$

$$\text{At } \omega = \omega_{c2} \text{ } A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A_{(\text{At } \omega = \omega_{c1})} = -54 \text{ db}$$

$$\text{At } \omega = \omega_h \text{ } A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{(\text{At } \omega = \omega_{c2})} = -102 \text{ db}$$

✓ **Step: 4 (3 Mark)**

Find phase angle:

$$\varphi = -0.2\omega \times \frac{180}{\pi} - 90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 0.125\omega$$

ω	φ
0.1	-94
0.5	-114
1	-134
2	-172
4	-226

✓ **Step: 5 (5 Mark)**

Draw the bode plot:

ω	Gain in db	φ
0.1	-	-94
0.5	-18	-114
1	-	-134
2	-30	-172
4	-	-226
8	-54	-
50	-102	-

Find K if Gain margin = 6 db:

With K=1, the gain margin = 26db

$$20 \log K = 26$$

$$K = 10^{-\left(\frac{26}{20}\right)} = 0.050$$

Find K if Phase margin = 45°:With K=1, $\varphi_{gc} = \gamma + 180^\circ = 45^\circ - 180^\circ = -135^\circ$ At $\varphi_{gc} = -135^\circ$ Gain = 24db

$$20 \log K = 24$$

$$K = 10^{-\left(\frac{24}{20}\right)} = 0.063$$

4. The open loop transfer function of a unity feedback system is given by $G(S) = \frac{1}{S(S+1)^2}$. Sketch the polar plot and determine the gain and phase margin. (Nov/Dec-16, May/Jun-14)

- Given: (2 Marks)

$$G(S) = \frac{1}{S(S+1)^2}$$

- Solution: (14 Marks)

- ✓ Step: 1 (1 Mark)

Sub $S=j\omega$

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)^2}$$

- ✓ Step: 2 (3 Marks)

Write magnitude and phase equation:

$$\text{Magnitude } M = \frac{1}{\omega(1+\omega^2)}$$

$$\text{Phase angle } \varphi = -90^\circ - 2\tan^{-1}\omega$$

- ✓ Step: 3 (3 Marks)

Calculate magnitude and phase equation:

ω	0	0.2	0.4	0.6	0.8	1	5
M	∞	4.81	2.2	1.22	0.76	0.5	0.01
φ	-90	-112	-134	-152	-167	-180	-247

- ✓ Step: 4 (3 Marks)

Draw the polar plot:

- ✓ Step: 5 (4 Marks)

Find gain margin, phase margin:

$$\text{Gain cross over frequency: } \varphi_{gco} = -155^\circ$$

$$\text{Phase cross over frequency: } |G(j\omega_{pco})| = 0.5$$

$$\text{Gain Margin: } K_g = \frac{1}{|G(j\omega_{pco})|} = \frac{1}{0.5} = 2$$

$$\text{Phase Margin: } \gamma = 180^\circ - 155^\circ = 25^\circ$$

5. The open loop transfer function of a unity feedback system is given by $G(S) = \frac{1}{S^2(1+S)(1+2S)}$. Sketch the polar plot and determine the gain margin and phase margin. (Nov/Dec-10, Nov/Dec-15)

- Given: (2 Marks)

$$G(S) = \frac{1}{S^2(1+S)(1+2S)}$$

- Solution: (14 Marks)

- ✓ Step: 1 (1 Mark)

$$\text{Sub } S=j\omega, G(j\omega) = \frac{1}{j\omega^2(1+j\omega)(1+2j\omega)}$$

✓ **Step: 2 (3 Marks)**

Write magnitude and phase equation:

$$\text{Magnitude } M = \frac{1}{\omega^2(\sqrt{\omega^2+1})(\sqrt{4\omega^2+1})}$$

$$\text{Phase angle } \varphi = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

✓ **Step: 3 (3 Marks)**

Calculate magnitude and phase equation:

ω	0	0.2	0.4	0.5	0.6	0.7	0.8	1
M	∞	22.76	4.5	2.5	1.5	1	0.6	0.3
φ	-180	-213	-240	-251	-261	-269	-277	-288

✓ **Step: 4 (3 Marks)**

Draw the polar plot:

✓ **Step: 5 (4 Marks)**

Find gain margin, phase margin:

$$\text{Gain cross over frequency: } \varphi_{gco} = -269^\circ$$

$$\text{Phase cross over frequency: } |G(j\omega_{pco})| = \infty$$

$$\text{Gain Margin: } K_g = \frac{1}{|G(j\omega_{pco})|} = \frac{1}{\infty} = 0$$

$$\text{Phase Margin: } \gamma = 180^\circ - 269^\circ = -89^\circ$$

6. The open loop transfer function of a unity feedback system is given by $G(s) = \frac{1}{s(1+s)(1+2s)}$.

Sketch the polar plot and determine the gain and phase margin. (Nov/Dec-14)

• **Given: (2 Marks)**

$$G(S) = \frac{1}{s(1+s)(1+2s)}$$

• **Solution: (14 Marks)**

✓ **Step: 1 (1 Mark)**

$$\text{Sub } S=j\omega \quad G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

✓ **Step: 2 (3 Marks)**

Write magnitude and phase equation:

$$\text{Magnitude } M = \frac{1}{\omega(\sqrt{\omega^2+1})(\sqrt{4\omega^2+1})}$$

$$\text{Phase angle } \varphi = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

✓ **Step: 3 (3 Marks)**

Calculate magnitude and phase equation:

ω	0	0.2	0.4	0.5	0.6	0.7	0.8	1	5
M	∞	4.55	1.8	1.2	0.9	0.7	0.5	0.3	0.003
φ	-90	-123	-150	-162	-171	-180	-187	-198	-253

✓ **Step: 4 (3 Marks)**

Draw the polar plot:

✓ **Step: 5 (4 Marks)**

Find gain margin, phase margin:

$$\text{Gain cross over frequency: } \phi_{gco} = -165^\circ$$

$$\text{Phase cross over frequency: } |G(j\omega_{pco})| = 0.7$$

$$\text{Gain Margin: } K_g = \frac{1}{|G(j\omega_{pco})|} = \frac{1}{0.7} = 1.43$$

$$\text{Phase Margin: } \gamma = 180^\circ - 165^\circ = -15^\circ$$

7. Plot the polar plot for the following transfer function $G(S) = \frac{15}{(1+S)(S+3)(S+6)}$ (Apr/May-17)

• **Given: (2 Marks)**

$$G(S) = \frac{15}{(1+S)(S+3)(S+6)}$$

• **Solution: (14 Marks)**✓ **Step: 1 (1 Mark)**

Sub $S=j\omega$

$$G(j\omega) = \frac{15}{(1+j\omega)(j\omega+3)(j\omega+6)}$$

✓ **Step: 2 (3 Marks)**

Write magnitude and phase equation:

$$\text{Magnitude } M = \frac{15}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+9})(\sqrt{\omega^2+36})}$$

$$\text{Phase angle } \phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{3} - \tan^{-1}\frac{\omega}{6}$$

✓ **Step: 3 (3 Marks)**

Calculate magnitude and phase equation:

ω	0	0.5	1	2	3	5	6	8
M	0.833	0.732	0.551	0.294	0.166	0.064	0.043	0.021
ϕ	0	-40.79	-72.89	-115.50	-143.13	-177.50	-188.90	-205.40

✓ **Step: 4 (3 Marks)**

Draw the polar plot:

✓ **Step: 5 (4 Marks)**

Find gain margin, phase margin:

$$\text{Gain cross over frequency: } \phi_{gco} = \infty$$

$$\text{Phase cross over frequency: } |G(j\omega_{pco})| = 5.5$$

$$\text{Gain Margin: } K_g = \frac{1}{|G(j\omega_{pco})|} = \frac{1}{5.5} = 0.18$$

$$\text{Phase Margin: } \gamma = 180^\circ - \phi_{gco} = \infty$$

8. For the unity feedback system with closed loop transfer function $G(S) / (1+G(S))$. Derive the equations for the locus of constant M and N circles. (May/Jun-12, Nov/Dec-14)

• **Constant M circles: (8 Marks)**

The magnitude of closed loop transfer function with unit feedback can be shown for every value of M. These circles are called M circles.

$$M(S) = \frac{G(S)}{1+G(S)}$$

Put $s=j\omega$, $M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$

Let, $G(j\omega)=X + jY$

$$M(j\omega) = \frac{X + jY}{1+X + jY} = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}} \angle \left(\tan^{-1} \frac{Y}{X} + \tan^{-1} \frac{Y}{1+X} \right)$$

Let M=Magnitude of M (j ω)

$$M = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}$$

Squaring on both sides, $M = \frac{X^2+Y^2}{(1+X)^2+Y^2}$

$$X^2(M^2 - 1) + M^2 2X + M^2 + Y^2(M^2 - 1) = 0$$

Divide by $(M^2 - 1)$,

$$X^2 + M^2 \frac{2X}{(M^2 - 1)} + \frac{M^2}{(M^2 - 1)} + Y^2 = 0$$

Add $\frac{M^2}{(M^2-1)^2}$ on both sides

$$X^2 + M^2 \frac{2X}{(M^2 - 1)} + \frac{M^2}{(M^2 - 1)} + Y^2 + \frac{M^2}{(M^2 - 1)^2} = \frac{M^2}{(M^2 - 1)^2}$$

$$\left[X + \frac{M^2}{M^2 - 1} \right]^2 + Y^2 = \frac{M^2}{(M^2 - 1)^2}$$

The equation of circle with centre at (X_1, Y_1) and radius r is given by,

$$(X - X_1)^2 + (Y - Y_1)^2 = r^2$$

When $M=0$

$$X_1 = 0$$

$$Y_1 = 0$$

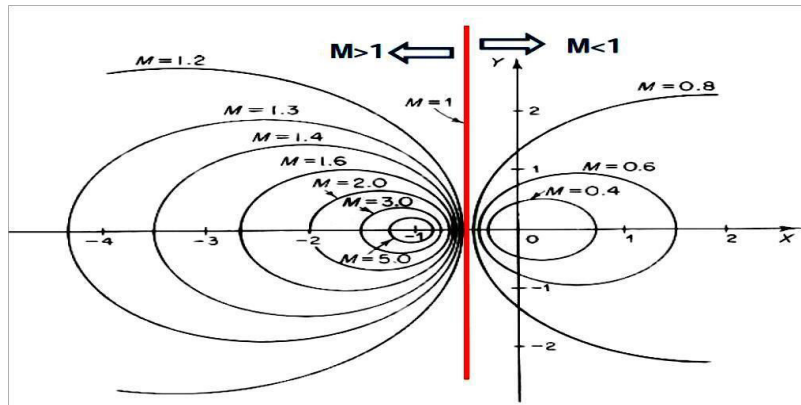
$$r = 0$$

When $M=\infty$

$$X_1 = -1$$

$$Y_1 = 0$$

$$r = 0$$



• **Constant N circles: (8 Marks)**

If the phase of closed loop transfer function with unity feedback is α , then it can be shown that $\tan \alpha$ will be in the form of circle for every value of α . These circles are called N circles.

$$M(S) = \frac{G(S)}{1+G(S)}$$

Put $s=j\omega$, $M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$

Let, $G(j\omega)=X + jY$

$$M(j\omega) = \frac{X + jY}{1+X + jY} = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}} \angle \left(\tan^{-1} \frac{Y}{X} + \tan^{-1} \frac{Y}{1+X} \right)$$

Let α =phase of $M(j\omega)$ $\alpha = \tan^{-1} \frac{Y}{X} + \tan^{-1} \frac{Y}{1+X}$

Let $N = \tan \alpha$ $N = \tan \left(\tan^{-1} \frac{Y}{X} + \tan^{-1} \frac{Y}{1+X} \right)$

$$N = \frac{\tan\left(\tan^{-1} \frac{Y}{X}\right) - \tan\left(\tan^{-1} \frac{Y}{1+X}\right)}{1 + \tan\left(\tan^{-1} \frac{Y}{X}\right) \tan\left(\tan^{-1} \frac{Y}{1+X}\right)}$$

$$N = \frac{Y}{X + X^2 + Y^2}$$

On rearranging above equation

$$X + X^2 + Y^2 - \frac{Y}{N} = 0$$

Add $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$ on both sides

$$X + X^2 + Y^2 - \frac{Y}{N} + \frac{1}{4} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left[X + \frac{1}{2}\right]^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

The equation of circle with centre at (X_1, Y_1) and radius r is given by,

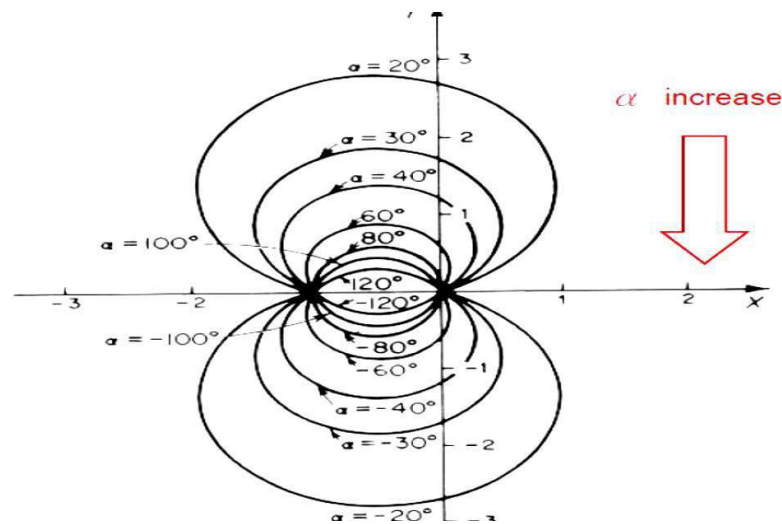
$$(X - X_1)^2 + (Y - Y_1)^2 = r^2$$

When $X=0, Y=0$

$$\frac{1}{4} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

When $X=-1, Y=0$

$$\frac{1}{4} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$



9. Write down the procedure for designing lead compensator using bode plot. (May/Jun-16, May/Jun-13)

- ✓ Determine K from the error constants given
- ✓ Sketch the bode plot
- ✓ Determine phase margin γ
- ✓ Find the amount of phase angle to be contributed by lead network is,

$$\varphi_m = \gamma_d - \gamma + \epsilon$$

Where, γ_d is the required phase margin

ϵ is 5 initially.

If the angle is greater than 60 then we have to design the compensator as 2 cascade compensator with lead angle as $\frac{\varphi_m}{2}$

- ✓ calculate $\alpha = \frac{1 - \sin \varphi_m}{1 + \sin \varphi_m}$ from bode plot find ω_m such that it is the frequency corresponding to the gain $-20 \log \frac{1}{\sqrt{\alpha}}$
- ✓ calculate $T = \frac{1}{\omega_m \sqrt{\alpha}}$ a lead compensator has the form $\frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$
- ✓ Find the complete transfer function with the lead compensator added in series to the original system
- ✓ plot the new Bode plot and determine phase margin and observe that it is the required phase margin if not satisfactory repeat steps from step 4 by changing value of ϵ by 5

10. Discuss in detail about lag, lead and lead-lag compensators with examples. (Apr/May-17)

Lead Compensator:

- ✓ **Step 1:** Determine K from the error constants given
- ✓ **Step 2:** Sketch the bode plot
- ✓ **Step 3:** Determine phase margin γ

- ✓ **Step 4:** Find the amount of phase angle to be contributed by lead network is,

$$\varphi_m = \gamma_d - \gamma + \epsilon$$

Where, γ_d is the required phase margin

ϵ is 5 initially.

If the angle is greater than 60 then we have to design the compensator as 2 cascade compensator with lead angle as $\frac{\varphi_m}{2}$

- ✓ **Step 5:** Calculate $\alpha = \frac{1 - \sin \varphi_m}{1 + \sin \varphi_m}$ from bode plot find ω_m such that it is the frequency corresponding to the gain $-20 \log \frac{1}{\sqrt{\alpha}}$
- ✓ **Step 6:** Calculate $T = \frac{1}{\omega_m \sqrt{\alpha}}$ a lead compensator has the form $\frac{S + \frac{1}{T}}{S + \frac{1}{\alpha T}}$
- ✓ **Step 7:** Find the complete transfer function with the lead compensator added in series to the original system
- ✓ **Step 8:** Plot the new Bode plot and determine phase margin and observe that it is the required phase margin if not satisfactory repeat steps from step 4 by changing value of ϵ by 5

Lag Compensator:

- ✓ **Step 1:** Determine K from the error constants given
- ✓ **Step 2:** Sketch the bode plot
- ✓ **Step 3:** Determine phase margin γ
- ✓ **Step 4:** Find the amount of phase angle to be contributed by lead network is,

$$\gamma_n = \gamma_d + \epsilon$$

Where, γ_d is the required phase margin

ϵ is 5 initially.

- ✓ **Step 5:** Find new gain cross over frequency ω_{gcn} corresponding to phase margin γ_n on the bode plot of uncompensated system.

$$\gamma_n = 180^\circ + \varphi_{gcn}$$

- ✓ **Step 6:** Find β of the compensator.

$$A_{gcn} = 20 \log \beta$$

$$\beta = 10^{A_{gcn}/20}$$

- ✓ **Step 7:** Find transfer function: zeros $z_c = \frac{1}{T} = \frac{\omega_{gcn}}{10}$, poles $P_c = \frac{1}{\beta T}$

$$\text{Transfer function } G_c(s) = \beta \left[\frac{1+sT}{1+\beta sT} \right]$$

- ✓ **Step 8:** Find transfer function $G_o(s) = \beta \left[\frac{1+sT}{1+\beta sT} \right] G(s)$

- ✓ **Step 9:** Determine actual phase margin $\gamma_0 = 180^\circ + \varphi_{gcn}$.

If actual phase margin satisfies the given specification, then the design is accepted. Otherwise repeat the procedure from 4 to 9 by increasing ϵ as 5 more than previous design.

Lead-Lag Compensators:

✓ **Step 1:** Determine K from the error constants given

✓ **Step 2:** Sketch the bode plot

✓ **Step 3:** Determine phase margin $\gamma = 180^\circ + \varphi_{gcn}$

✓ **Step 4:** Choose new phase margin as $\gamma_n = \gamma_d + \epsilon$

Where, γ_d is the required phase margin

ϵ is 5 initially.

✓ **Step 5:** Find new gain cross over frequency ω_{gcn} corresponding to phase margin γ_n on the bode plot of uncompensated system. $\gamma_n = 180^\circ + \varphi_{gcn}$

✓ **Step 6:** Find β of the lag compensator.

$$A_{gcl} = 20 \log \beta$$

$$\beta = 10^{A_{gcl}/20}$$

✓ **Step 7:** Find transfer function of lag section: zeros $z_{cl} = \frac{1}{T_1} = \frac{\omega_{gcl}}{10}$, poles $P_{cl} = \frac{1}{\beta T_1}$

Transfer function of lag compensator $G_1(s) = \beta \left[\frac{1+sT_1}{1+\beta sT_1} \right]$

✓ **Step 8:** Find transfer function of lead section, take $\alpha = \frac{1}{\beta}$

✓ **Step 9:** Find transfer function of lead compensator $G_2(s) = \alpha \left[\frac{1+sT_2}{1+\alpha sT_2} \right]$

✓ **Step 10:** Find transfer function of lag-lead compensator $G_c(s) = G_1(s) \times G_2(s)$

$$G_c(s) = \frac{(1 + sT_1)(1 + sT_2)}{(1 + \beta sT_1)(1 + \alpha sT_2)}$$

✓ **Step 11:** Find open loop transfer function $G_o(s) = G_c(s) \times G(s)$

✓ **Step 12:** Determine actual phase margin $\gamma_0 = 180^\circ + \varphi_{gcn}$.

Plot the new Bode plot and verify specification is satisfied or not. If not satisfied, then choose another choice of α , such that $\alpha < \frac{1}{\beta}$ and repeat steps from step 9 to 12.

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EC6405 - CONTROL SYSTEM ENGINEERING

QUESTION BANK

UNIT - IV STABILITY ANALYSIS

PART -A

1. Define stability and what is the necessary condition for stability? (May/Jun-11, Nov/Dec-11)

A linear relaxed system is said to have BIBO stability if every bounded input results in a bounded output.

The necessary condition for stability is that all the coefficients of the characteristic polynomial be positive. The necessary and sufficient condition for stability is that all of the elements in the first column of the routh array should be positive.

2. State Nyquist stability criterion. (Apr/May-17, May/Jun-16, Nov/Dec-15, Nov/Dec-13, May/Jun-13, May/Jun-12, Nov/Dec-12, May/Jun-10)

If the Nyquist plot of the open loop transfer function $G(s)$ corresponding to the Nyquist control in the S-plane encircles the critical point $-1+j0$ in the counter clockwise direction as many times as the number of right half S-plane poles of $G(s)$, the closed loop system is stable.

3. What is Routh stability criterion? (Apr/May-17, May/Jun-13)

Routh criterion states that the necessary and sufficient condition for stability is that all of the elements in the first column of the routh array is positive. If this condition is not met, the system is unstable and the number of sign changes in the elements of the first column of routh array corresponds to the number of roots of characteristic equation in the right half of the S-plane.

4. Define Relative stability. (May/Jun-14)

Relative stability is the degree of closeness of the system, it is an indication of strength or degree of stability.

5. What are root loci? (May/Jun-12)

The path taken by the roots of the open loop transfer function when the loop gain is varied from 0 to ∞ are called root loci.

6. What is a dominant pole? (Nov/Dec-16, Nov/Dec-15, Nov/Dec-14)

The dominant pole is a complex conjugate pair which decides the transient response of the system.

7. What is limitedly stable system?

For a bounded input signal if the output has constant amplitude oscillations, then the system may be stable or unstable under some limited constraints such a system is called limitedly stable system.

8. What are asymptotes? How will you find angle of asymptotes?

Asymptotes are the straight lines which are parallel to root locus going to infinity and meet the root locus at infinity.

Angles of asymptotes = $\pm 180^\circ(2q + 1)/(n-m)$ $q= 0,1,2, \dots,(n-m)$

n-number of poles, m-number of zeros.

9. What is centroid?

The meeting point of the asymptotes with the real axis is called centroid. The centroid is given by

Centroid = (sum of poles – sum of zeros) / (n-m)

n-number of poles. m-number of zeros.

10. What is angle criterion?

The angle criterion states that $s=s_a$ will be the point on the root locus if for that value of S the argument or phase of $G(S)H(S)$ is equal to an odd multiple of 180° .

(Sum of the angles of vectors from zeros to the point $s=s_a$)- (Sum of the angles of vectors from poles to the point $s=s_a$) = $\pm 180^\circ(2q + 1)$

PART - B (Answer as Hint)

1. Construct Routh array and determine the stability of the system whose characteristics equation is $S^6+2S^5+8S^4+12S^3+20S^2+16S+16 = 0$. (May/Jun-16, May/Jun-13, May/Jun-12, Nov/Dec-14)

- Given: (2 Marks)

$S^6+2S^5+8S^4+12S^3+20S^2+16S+16 = 0$

- Solution: (14 Marks)

✓ Draw routh array: (8 Marks)

S^6	1	8	20	16
S^5	2 1	12 6	16 8	
S^4	1	6	8	
S^3	0 3 1	0 12 4		
S^2	3	8		
S^1	0.33			
S^0	8			

$A = S^4 + 6S^2 + 8$
 $\frac{dA}{dS} = 3S^3 + 12S$

Column 1

✓ Find roots: (4 Marks)

$S^4 + 6S^2 + 8 = 0$

Let, $S^2 = x$

$x^2 + 6x + 8 = 0$

$x = -2 \text{ or } -4$

$$S = \pm\sqrt{x}$$

$$S = \pm\sqrt{-2} \text{ or } \pm\sqrt{-4}$$

$$S = +j\sqrt{2}, -j\sqrt{2}, +j2, -j2$$

Totally 6 roots.

✓ **Result: (2 Marks)**

- System is marginally stable.
- 4 roots are lying on imaginary axis, remaining 2 roots are lying on left half of S-plane because no sign changes in column 1.

2. By using Routh criterion determine the stability of the system represented by the following characteristics equation $S^5 + S^4 + 2S^3 + 2S^2 + 11S + 10$ (May/Jun-14)

• **Given: (2 Marks)**

$$S^5 + S^4 + 2S^3 + 2S^2 + 11S + 10 = 0.$$

• **Solution: (14 Marks)**

✓ **Draw routh array: (6 Marks)**

S^5	1	2	11
S^4	1	2	10
S^3	0 ϵ	1	
S^2	$\frac{2\epsilon - 1}{\epsilon}$	10	
S^1	$\frac{-(10\epsilon^2 - 2\epsilon + 1)}{2\epsilon - 1}$		
S^0	10		

✓ **On letting $\epsilon \rightarrow 0$, we get: (5 Marks)**

S^5	1	2	11
S^4	1	2	10
S^3	0	1	
S^2	$-\infty$	10	
S^1	1		
S^0	10		

↑ Column 1

✓ **Find roots: (1 Marks)**

Totally 5 roots.

✓ **Result: (2 Marks)**

- System is unstable.
- 2 roots are lying on right half of s-plane because 2 sign changes in column 1 and remaining 3 roots are lying on left half of S-plane.

3. Using Routh criterion, determine the stability of the system represented by the characteristic equation $S^5 + S^4 + 2S^3 + 2S^2 + 3S + 5 = 0$. Comment on the location of the roots of characteristics equation. (Nov/Dec-16, Nov/Dec-15)

- Given: (2 Marks)

$$S^5 + S^4 + 2S^3 + 2S^2 + 3S + 5 = 0.$$

- Solution: (14 Marks)

✓ Draw routh array: (6 Marks)

S^5	1	2	3
S^4	1	2	5
S^3	0 ϵ	-2	
S^2	$\frac{2\epsilon - 2}{\epsilon}$	5	
S^1	$\frac{-(5\epsilon^2 + 4\epsilon + 4)}{2\epsilon + 2}$		
S^0	5		

✓ On letting $\epsilon \rightarrow 0$, we get: (5 Marks)

S^5	1	2	3
S^4	1	2	5
S^3	0	-2	
S^2	∞	5	
S^1	-2		
S^0	5		

↑ Column 1

✓ Find roots: (1 Marks)

Totally 5 roots.

✓ Result: (2 Marks)

- System is unstable.
- 2 roots are lying on right half of s-plane because 2 sign changes in column 1 and remaining 3 roots are lying on left half of S-plane.

4. Draw the Nyquist plot and find the stability of the following open loop transfer function of unity feedback control system $G(S)H(S) = K(S + 1)/S^2(S + 10)$. If the system is conditionally stable, find the range of K for which the system is stable. (May/Jun-15)

- Given: (2 Marks)

$$G(S)H(S) = K(S + 1)/S^2(S + 10)$$

- **Solution: (14 Marks)**

- ✓ **Find characteristics equation: (4 Marks)**

$$GG(S)H(S) = K(S + 1)/S^2(S + 10)$$

$$\text{Characteristics equation: } 1 + G(S)H(S) = 0$$

$$S^3 + 10S^2 + KS + K = 0$$

- ✓ **Draw routh array: (6 Marks)**

S^3	1	K	
S^2	10	K	
S^1	$\frac{9K}{10}$		
S^0	K		

- ✓ **Find K: (4 Marks)**

$$K > 0$$

$$\frac{9K}{10} > 0$$

$$9K > 10$$

$$K > \frac{10}{9}, \quad K < \frac{9}{10}$$

$$\text{Range of K} = 0 < K < \frac{9}{10}$$

5. Determine the range of K for stability of unity feedback system using Routh stability criterion whose transfer function is $\frac{C(S)}{R(S)} = \frac{K}{S(S^2+S+1)(S+2)+K}$ (Apr/May-17)

- **Given: (2 Marks)**

$$\frac{C(S)}{R(S)} = \frac{K}{S(S^2 + S + 1)(S + 2) + K}$$

- **Solution: (14 Marks)**

- ✓ **Find characteristics equation: (4 Marks)**

$$G(S)H(S) = \frac{K}{S(S^2 + S + 1)(S + 2)}$$

$$\text{Characteristics equation: } 1 + G(S)H(S) = 0$$

$$S^4 + 3S^3 + 3S^2 + 2S + K = 0$$

- ✓ **Draw routh array: (6 Marks)**

S^4	1	3	K
S^3	3	2	
S^2	2.33	K	
S^1	$\frac{4.66 - 3K}{2.33}$	0	
S^0	K		

✓ Find K: (4 Marks)

$$\frac{4.66 - 3K}{2.33} > 0$$

$$4.66 - 3K > 0$$

$$4.66 > 3K$$

$$K < 1.553$$

$$\text{Range of } K = 0 < K < 1.553$$

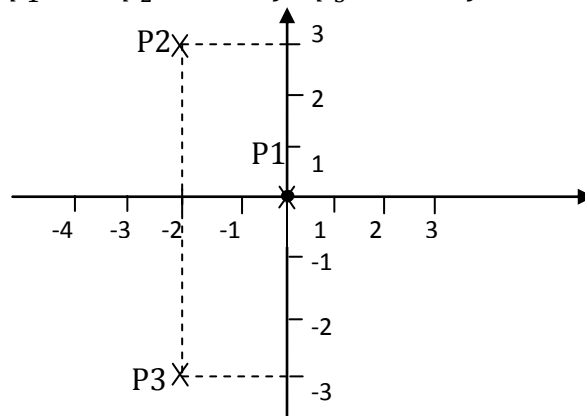
6. Sketch the root locus for the open loop transfer function of unity feedback control system given below. $G(s) = k/[s(s^2 + 4s + 13)]$ (May/Jun-16, Nov/Dec-14)

• Step 1: (2 Mark)

Find poles and zeros and Locate poles and zeros.

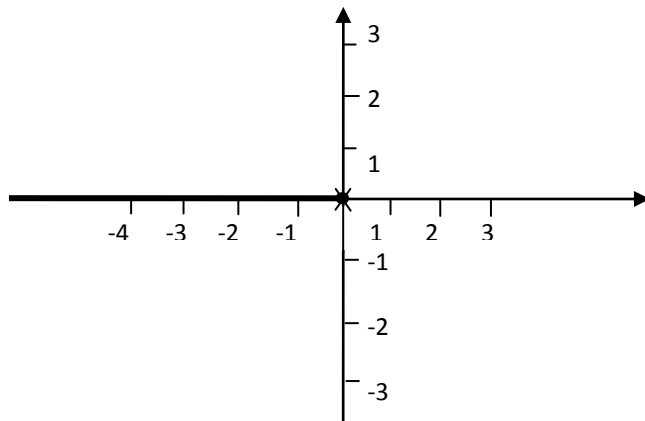
Number of poles $n=3$, Number of zeros $m=0$

$$p_1 = 0, p_2 = -2 + j3, p_3 = -2 - j3$$



• Step 2: (2 Mark)

Find root locus on real axis.



• Step 3: (2 Marks)

Find angle of asymptotes and centroid.

$$n - m = 3 - 0 = 3$$

$$\text{Angles of asymptotes} = \pm \frac{180(2q+1)}{n-m} \quad q = 0, 1, 2$$

$$\text{if } q=0, \text{ angle}=60$$

$$\text{if } q=1, \text{ angle}=180$$

$$\text{if } q=2, \text{ angle}=300$$

Centroid = (sum of poles - sum of zeros) / (n-m)

$$= [(0-2+j3-2-j3)-0]/3 = -1.33$$

• **Step 4: (2 Marks)**

Find break away:

Let characteristics equation $1 + G(S)H(S) = 0$

$$1 + \frac{K}{S(S^2 + 4S + 13)} = 0$$

$$K = -(S^3 + 4S^2 + 13S)$$

$$\frac{dK}{dS} = -(3S^2 + 8S + 13)$$

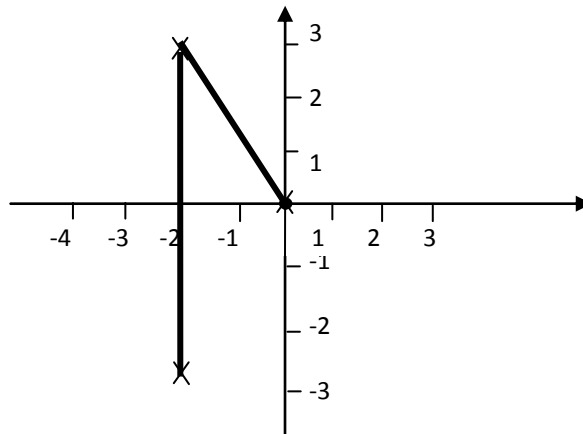
$$S = -1.33 \pm j1.6$$

Select break away point between 0 and ∞ .

If $S = -1.33 \pm j1.6$, then the value is not equal to zero. So no break away point.

• **Step 5: (2 Marks)**

Find angle of departure.



$$\theta_1 = 123.7^\circ, \theta_2 = 90^\circ$$

$$\text{Angle of departure at pole P2} = 180 - (\theta_1 + \theta_2) = -33.7^\circ$$

$$\text{Angle of departure at pole P3} = 33.7^\circ$$

• **Step 6: (2 Marks)**

Find crossing point on imaginary axis.

The characteristics equation is

$$S^3 + 4S^2 + 13S + K = 0$$

$$\text{Solve it and, } K = 52, \omega = \pm 3.6$$

• **Step 7: (4 Marks)**

Draw the root locus on graph.

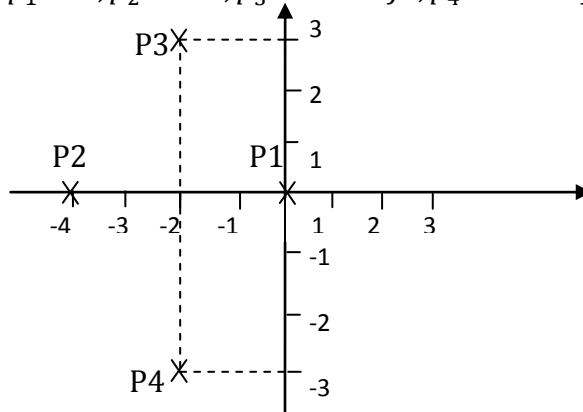
7. Draw the root locus plot for the system whose open loop transfer function is given by $G(S)H(S) = K/S(S + 4)(S^2 + 4S + 13)$. find the marginal value of k which causes sustained oscillations and the frequency of oscillations. (May/Jun-14)

- **Step 1: (2 Mark)**

Find poles and zeros and Locate poles and zeros.

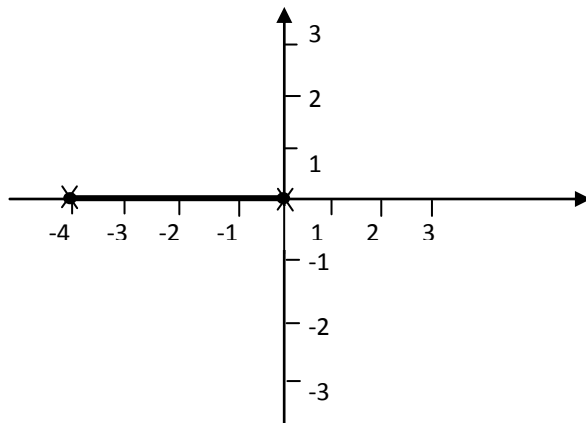
Number of poles $n=4$, Number of zeros $m=0$

$$p_1 = 0, p_2 = -4, p_3 = -2 + j3, p_4 = -2 - j3$$



- **Step 2: (2 Mark)**

Find root locus on real axis.



- **Step 3: (2 Marks)**

Find angle of asymptotes and centroid.

$$n-m = 4-0=4$$

$$\text{Angles of asymptotes} = \pm \frac{180(2q+1)}{n-m} \quad q=0,1,2,3$$

$$\text{if } q=0, \text{ angle}=45$$

$$\text{if } q=1, \text{ angle}=135$$

$$\text{if } q=2, \text{ angle}=225$$

$$\text{if } q=3, \text{ angle}=315$$

$$\text{Centroid} = (\text{sum of poles} - \text{sum of zeros}) / (n-m) = [(0-4-2+j3-2-j3)-0]/4 = -2$$

- **Step 4: (2 Marks)**

Find break away:

$$\text{Let characteristics equation } 1 + G(S)H(S) = 0$$

$$1 + \frac{K}{S(S+4)(S^2+4S+13)} = 0$$

$$K = -(S^4 + 8S^3 + 29S^2 + 52S)$$

$$\frac{dK}{dS} = -(4S^3 + 24S^2 + 58S + 52)$$

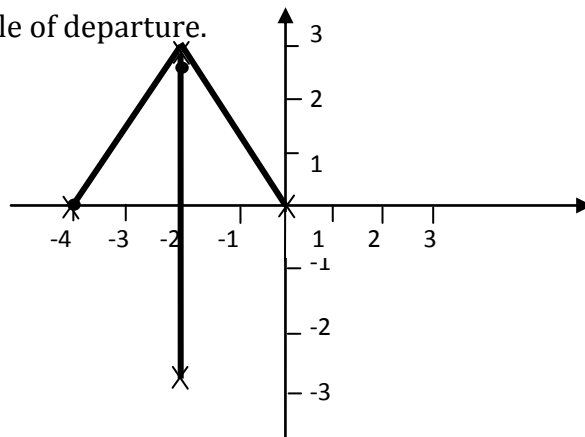
Select break away point between 0 and -4.

If $S=-2$, then the value is zero. So break away point is -2.

And complex break away points are $-2 + j1.58, -2 - j1.58$

• **Step 5: (2 Marks)**

Find angle of departure.



$$\theta_1 = 124^\circ, \theta_2 = 50^\circ, \theta_4 = 90^\circ$$

$$\text{Angle of departure at pole } P_4 = 180 - (\theta_1 + \theta_2 + \theta_4) = -84^\circ$$

$$\text{Angle of departure at pole } P_3 = 84^\circ$$

• **Step 6: (2 Marks)**

Find crossing point on imaginary axis.

The characteristics equation is

$$S^4 + 8S^3 + 29S^2 + 52S + K = 0$$

$$\text{Solve it and, } K = 146.25, \omega = \pm 2.55$$

• **Step 7: (4 Marks)**

Draw the root locus on graph.

8. The open loop transfer function of unity feedback system is given by $G(S) = \frac{K(S+9)}{S(S^2+4S+11)}$.

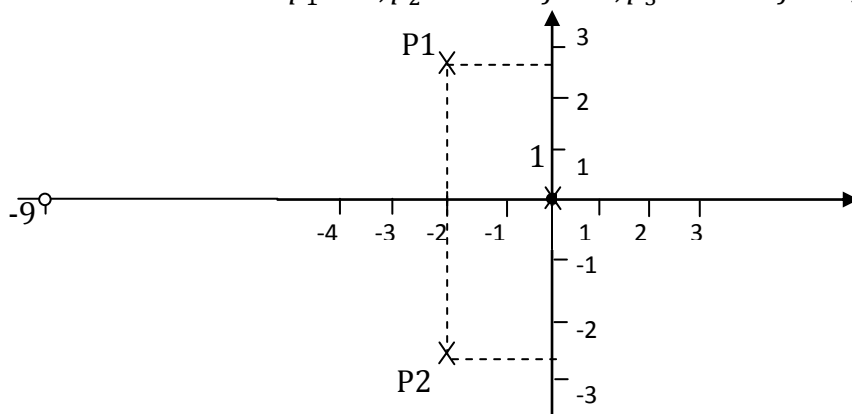
Sketch the root locus of the system. (Nov/Dec-15)

• **Step 1: (2 Mark)**

Find poles and zeros and Locate poles and zeros.

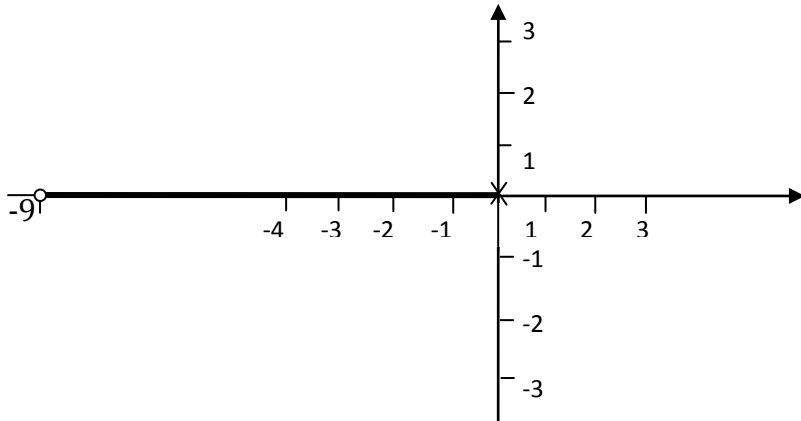
Number of poles $n=3$, Number of zeros $m=1$

$$p_1 = 0, p_2 = -2 + j2.64, p_3 = -2 - j2.64, z_1 = -9$$



- **Step 2: (2 Mark)**

Find root locus on real axis.



- **Step 3: (2 Marks)**

Find angle of asymptotes and centroid.

$$n-m = 3-1=2$$

$$\text{Angles of asymptotes} = \pm \frac{180(2q+1)}{n-m} \quad q=0,1,2$$

$$\text{if } q=0, \text{ angle}=90$$

$$\text{if } q=1, \text{ angle}=180$$

$$\text{if } q=2, \text{ angle}=450$$

$$\text{Centroid} = (\text{sum of poles} - \text{sum of zeros}) / (n-m)$$

$$= [(0-2+j2.64-2-j2.64)-(-9)]/2 = 2.5$$

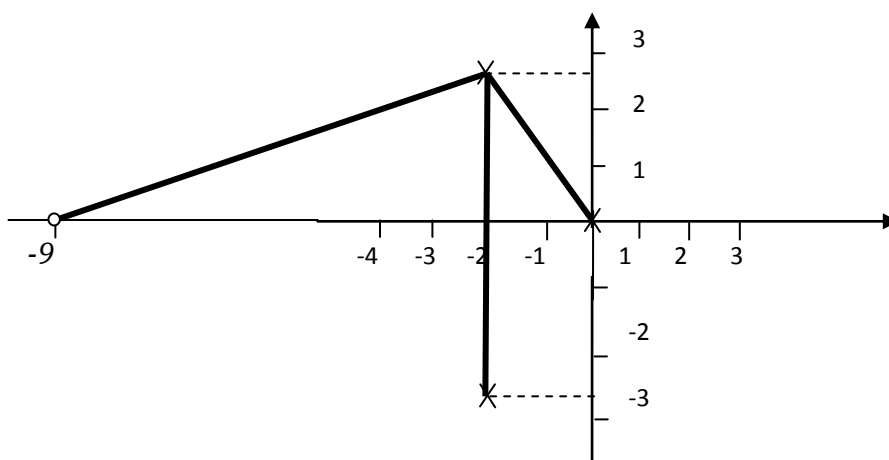
- **Step 4: (2 Marks)**

Find break away:

There is no possibility of break away point or break in point because root locus located between poles and zeros.

- **Step 5: (2 Marks)**

Find angle of departure.



$$\theta_1 = 127.1^\circ, \theta_2 = 90^\circ, \theta_3 = 20.7^\circ$$

$$\text{Angle of departure at pole } P_2 = 180 - (\theta_1 + \theta_2) + \theta_3 = -16.4^\circ$$

$$\text{Angle of departure at pole } P_3 = 16.4^\circ$$

- **Step 6: (2 Marks)**

Find crossing point on imaginary axis.

The characteristics equation is

$$(S^3 + 4S^2 + 11S) + KS + 9K = 0$$

$$\text{Solve it and, } K = 8.8, \omega = \pm 4.4$$

- **Step 7: (4 Marks)**

Draw the root locus on graph.

9. With neat steps write down the procedure for construction of root locus. Each rule gives an example. (Nov/Dec-16, Apr/May-17)

- **Step 1: (2 Mark)**

Find poles and zeros and Locate poles and zeros.

- **Step 2: (2 Mark)**

Find root locus on real axis.

- **Step 3: (2 Marks)**

Find angle of asymptotes and centroid.

- Angles of asymptotes = $\pm 180^\circ(2q + 1)/(n-m)$ $q = 0, 1, 2, \dots, (n-m)$

- Centroid = $(\text{sum of poles} - \text{sum of zeros}) / (n-m)$

- **Step 4: (2 Marks)**

Find break away and break in point.

Let characteristics equation $B(S)+KA(S)=0$

$$K = -B(S)/A(S)$$

$$\frac{dK}{dS} = 0$$

- **Step 5: (2 Marks)**

Find angle of departure and angle of arrival.

For example:

- If poles are complex, then Angle of departure at pole $P_1 = 180 - (\theta_1 + \theta_3) + (\theta_2 + \theta_4)$

- If zeros are complex, then Angle of arrival at pole $P_1 = 180 - (\theta_1 + \theta_3) + (\theta_2 + \theta_4)$

- **Step 6: (2 Marks)**

Find crossing point on imaginary axis.

- **Step 7: (4 Marks)**

Draw the root locus on graph.

10. Draw the root locus diagram for a system open loop transfer function and then determine the value of K such that the damping ratio of the dominant closed loop poles is 0.4. Open loop transfer function = $\frac{20}{s(s+1)(s+4)+20KS}$. (Apr/May-17)

- **Given: (1 Marks)**

$$G(s) = \frac{20}{s(s+1)(s+4)+20KS}$$

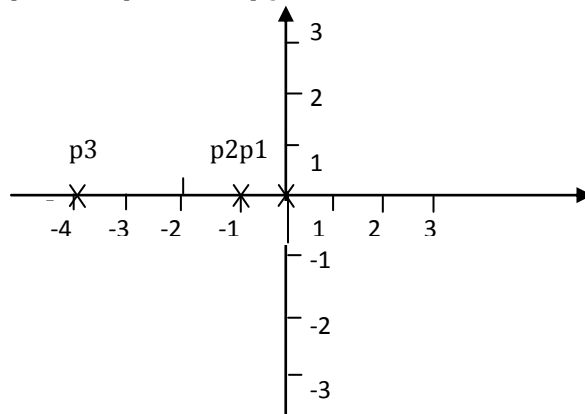
$$\zeta = 0.4$$

- **Step 1: (2 Mark)**

Find poles and zeros and Locate poles and zeros.

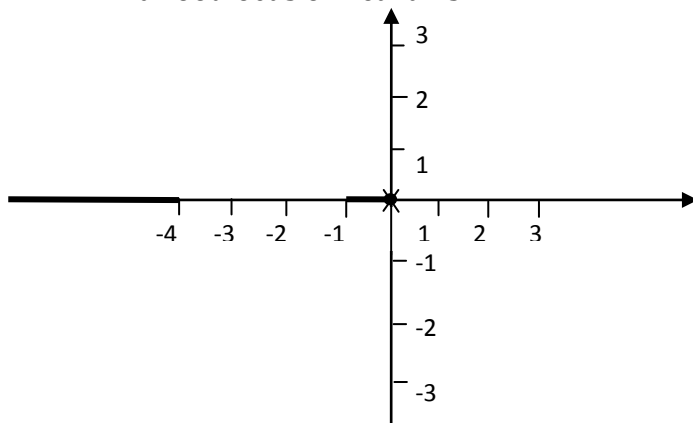
Number of poles $n=3$, Number of zeros $m=0$

$$p_1 = 0, p_2 = -1, p_3 = -4$$



- **Step 2: (2 Mark)**

Find root locus on real axis.



- **Step 3: (2 Marks)**

Find angle of asymptotes and centroid.

$$n-m = 3-0=3$$

$$\text{Angles of asymptotes} = \pm \frac{180(2q+1)}{n-m} \quad q=0,1,2$$

$$\text{if } q=0, \text{ angle}=60$$

$$\text{if } q=1, \text{ angle}=180$$

$$\text{if } q=2, \text{ angle}=300$$

$$\text{Centroid} = (\text{sum of poles} - \text{sum of zeros}) / (n-m)$$

$$= [(0-2-4)-0]/3 = -2$$

- **Step 4: (2 Marks)**

Find break away:

Let characteristics equation $1 + G(S)H(S) = 0$

$$1 + \frac{20}{s(s+1)(s+4) + 20KS} = 0$$

$$S = -2.21, 1.81$$

Select break away point between 0 and -1, -4 and ∞ . No break away point.

- **Step 5: (1 Marks)**

Find angle of departure.

There is no complex pole or zeros. So no angle of departure or angle of arrival.

- **Step 6: (2 Marks)**

Find crossing point on imaginary axis.

The characteristics equation is

$$S(S+1)(S+4) + 20KS + 20 = 0$$

$$\text{Solve it and, } K = 0, \omega = \pm 2$$

- **Step 7: (4 Marks)**

Draw the root locus on graph.

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EC6405 - CONTROL SYSTEM ENGINEERING

QUESTION BANK

UNIT - V STATE VARIABLE ANALYSIS

PART - A

1. State sampling theorem. (Apr/May-17, Nov/Dec - 16, May/Jun - 15, Nov/Dec - 10)

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency $F_s \geq 2F_{max}$ where F_s is the sampling frequency and F_{max} is the maximum frequency present in the signal.

2. What are hold circuits & explain it. (May/Jun - 15)

The function of the hold circuit is to reconstruct the signal which is applied as input to the sampler. The simplest holding device holds the signal between two consecutive instants at its preceded value till next sampling instant is reached.

3. What are the advantages of state space analysis?

It can be applied to non-linear as well as time varying systems. Any type of input can be considered for designing the system. It can be conveniently applied to multiple input multiple output systems. The state variables selected need not necessarily be the physical quantities of the system.

4. Define state variable. (May/Jun-16, May/Jun-13, Nov/Dec - 15, Nov/Dec-12)

The state of a dynamical system is a minimal set of variables (known as state variables) such that the knowledge of these variables at $t-t_0$ together with the knowledge of the inputs for $t > t_0$, completely determines the behavior of the system for $t > t_0$.

5. What is Controllability? (May/Jun - 15, Nov/Dec - 13)

A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $X(t_0)$ at any other desired state $X(t)$, in specified finite time by a control vector $U(t)$.

6. What is Observability? (May/Jun - 15)

A system is said to be completely observable if every state $X(t)$ can be completely identified by measurements of the output $Y(t)$ over a finite time interval.

7. Write the properties of state transition matrix. (Nov/Dec-16, Nov/Dec -15, May/Jun-14, May/Jun-10)

The following are the properties of state transition matrix

$$\phi(0) = e^{Ax0} = I \text{ (unit matrix).}$$

$$\phi(t) = e^{At} = (e^{-At})^{-1} = [\Phi(-t)]^{-1}.$$

$$\phi(t_1+t_2) = e^{A(t_1+t_2)} = \Phi(t_1) \Phi(t_2) = \Phi(t_2) \Phi(t_1).$$

8. What is sampled data control system? (Apr/May-17, Nov/Dec - 12)

When the signal or information at any or some points in a system is in the form of discrete pulses, then the system is called discrete data system or sampled data system.

9. What is the need for controllability test?

The controllability test is necessary to find the usefulness of a state variable. If the state variables are controllable then by controlling (i.e. varying) the state variables the desired outputs of the system are achieved.

10. What is the need for Observability test?

The Observability test is necessary to find whether the state variables are measurable or not. If the state variables are measurable then the state of the system can be determined by practical measurements of the state variables.

PART-B (Answer as Hint)**1. Obtain the state model of the system described by the following transfer function $\frac{y(S)}{u(S)} =$**

$$\frac{5}{s^2+6s+7} \quad (\text{May/Jun-14})$$

- **Given: (2 Marks)**

$$\frac{y(S)}{u(S)} = \frac{5}{s^2+6s+7}$$

- **Solution: (14 Marks)**

- ✓ **Find differential equation: (8 Marks)**

$$y(S)[s^2 + 6s + 7] = 5[u(S)]$$

$$y(S)s^2 + 6sy(S) + 7y(S) = 5u(S)$$

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 7y = 5u(t) \quad \text{----- (1)}$$

$$\text{Let } x_1 = y$$

$$x_2 = \dot{x}_1 = \frac{dy}{dt}, \quad x_3 = \dot{x}_2 = \frac{d^2y}{dt^2}$$

The eqn (1) becomes

$$\dot{x}_2 = -6x_2 - 7x_1 + 5u$$

- ✓ **Obtain state model: (6 Marks)**

State equation:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -7x_1 - 6x_2 + 5u$$

Output equation:

$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -7 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2. Obtain the transfer function model for the following state space system. $A = \begin{bmatrix} 1 & 1 \\ -6 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,

$C = [1 \ 0]$, $D = [0]$ (May/Jun - 14)

- Given: (2 Marks)

$$A = \begin{bmatrix} 1 & 1 \\ -6 & -5 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [1 \ 0], D = [0]$$

- Solution: (14 Marks)

- ✓ Obtain state model: (6 Marks)

The general equation is

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

State model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

- ✓ Find differential equation: (4 Marks)

From state model

$$\dot{x}_1 = x_1 + x_2 + u \quad \text{----- (1)}$$

$$\dot{x}_2 = -6x_1 - 5x_2 \quad \text{----- (2)}$$

$$y = x_1 \quad \text{----- (3)}$$

Obtain differential eqn.

$$y = x_1,$$

$$\dot{y} = \dot{x}_1 = x_2,$$

$$\ddot{y} = \dot{x}_2 = x_3$$

Eqn. (1), (2) & (3) becomes

$$y + u = 0$$

$$\dot{y} - 6u + 5\dot{y} = 0$$

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} = 6u(t) \quad \text{----- (4)}$$

- ✓ Find transfer function: (4 Marks)

Apply Laplace transform.

Eqn. (4) becomes

$$S^2Y(S) + 5SY(S) = 6U(S)$$

$$Y(S)[S^2 + 5S] = 6U(S)$$

$$T(S) = \frac{Y(S)}{U(S)} = \frac{6}{S^2 + 5S}$$

3. Consider the following system with differential equation given by $\ddot{y} + 6\dot{y} + 11y + 6y = 6u$ obtain the state model in diagonal canonical form. (Nov/Dec -15)

- Given: (2 Marks)

$$\ddot{y} + 6\dot{y} + 11y + 6y = 6u$$

- Solution: (14 Marks)

✓ Find differential equation: (8 Marks)

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = 6u(t) \text{ ----- (1)}$$

$$\text{Let } x_1 = y$$

$$x_2 = \dot{x}_1 = \frac{dy}{dt}, \quad x_3 = \dot{x}_2 = \frac{d^2y}{dt^2}, \quad x_4 = \dot{x}_3 = \frac{d^3y}{dt^3}$$

The eqn (1) becomes

$$\dot{x}_3 = -6x_3 - 11x_2 - 6x_1 + 6u$$

✓ Obtain state model: (6 Marks)

State equation:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + 6u$$

Output equation:

$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4. For a system represented by state equation $\dot{X}(t) = AX(t)$. The response is $X(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$ when $X(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $X(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ when $(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Determine the system matrix A and the state transition matrix. (Apr/May-17)

- Solution: (2 Marks)

The solution of state equation is, $X(t) = e^{At}X(0)$

Multiply above equation by e^{-At}

$$e^{-At}X(t) = X(0)$$

- Find state transition matrix: (8 Marks)

$$\text{Case (i): } X(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix} \text{ when } X(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$e^{-At} \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\text{Let, } e^{-At} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

$$\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\text{We get, } e_{11}e^{-2t} - 2e_{12}e^{-2t} = 1 \quad \text{----- (1)}$$

$$e_{21}e^{-2t} - 2e_{22}e^{-2t} = -2 \quad \text{----- (2)}$$

Case (ii): $X(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ When $X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$e^{-At} \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\text{We get, } e_{11}e^{-t} - e_{12}e^{-t} = 1 \quad \text{----- (3)}$$

$$e_{21}e^{-t} - e_{22}e^{-t} = -1 \quad \text{----- (4)}$$

Solve equation (1), (2), (3) & (4)

$$e_{11} = 2e^t - e^{2t}$$

$$e_{12} = e^t - e^{2t}$$

$$e_{21} = -2e^t + 2e^{2t}$$

$$e_{22} = -e^t + 2e^{2t}$$

So, state transition matrix is $e^{-At} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} = \begin{bmatrix} 2e^t - e^{2t} & e^t - e^{2t} \\ -2e^t + 2e^{2t} & -e^t + 2e^{2t} \end{bmatrix}$

- **Find system matrix: (6 Marks)**

$$\varphi(S) = L[e^{At}]$$

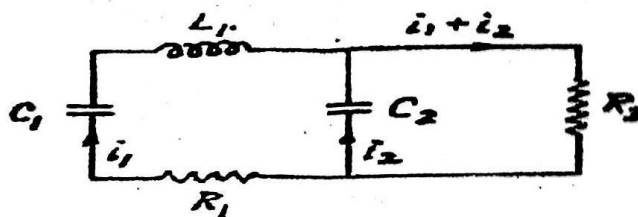
$$\varphi(S) = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$A = sI - \varphi(s)^{-1}$$

$$\varphi(s)^{-1} = \begin{vmatrix} s & -1 \\ 2 & s+3 \end{vmatrix}$$

$$\text{system matrix } A = \begin{vmatrix} 0 & 1 \\ -2 & -3 \end{vmatrix}$$

5. Construct a state model of the following electrical system. (Apr/May-17)



- **Step: 1 (3 Marks)**

Apply KVL for given electrical circuit

$$R_1 i_1 + L_1 \frac{di}{dt} + V_{c1} + V_{c2} = 0 \quad \text{----- (1)}$$

$$R_2 i_2 + V_{c2} = 0 \quad \text{----- (2)}$$

- **Step: 2 (3 Marks)**

Choose current through inductor $i_1 = X_1$

$$\text{Voltage across capacitor } V_{c1} = X_2 = \frac{1}{C_1} \int X_1 dt, \dots\dots (3)$$

$$V_{c2} = X_3 = \frac{1}{C_2} \int X_1 dt \dots\dots (4)$$

- **Step: 3 (3 Marks)**

Choose existing source as input u *No input source*

Choose Voltage drop across R_2 as output y

$$y_1 = R_1 i_1, y_2 = R_2 i_2$$

$$y_1 = R_1 X_1, y_2 = R_2 i_2$$

- **Step: 4 (4 Marks)**

Substitute state variable in KVL

$$\text{Eqn. (1) becomes } R_1 X_1 + L_1 \dot{X}_1 + X_2 + X_3 = 0$$

$$\dot{X}_1 = -\frac{R_1 X_1}{L_1} - \frac{X_2}{L_1} - \frac{X_3}{L_1}$$

$$\text{Eqn. (3) becomes } \dot{X}_2 = \frac{1}{C_1} X_1$$

$$\text{Eqn. (4) becomes } \dot{X}_3 = \frac{1}{C_2} X_1$$

- **Step: 5 (3 Marks)**

State model

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & -\frac{1}{L_1} & -\frac{1}{L_1} \\ \frac{1}{C_1} & 0 & 0 \\ \frac{1}{C_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}, [y_1] = [R_1][X_1]$$

6. Check the controllability of the following state space system $\dot{x}_1 = x_2 + u_2$; $\dot{x}_2 = x_3$;

$$\dot{x}_3 = -2x_2 - 3x_3 + u_1 + u_2. \text{ (May/Jun - 14)}$$

- **Given: (2 Marks)**

$$\dot{x}_1 = x_2 + u_2;$$

$$\dot{x}_2 = x_3;$$

$$\dot{x}_3 = -2x_2 - 3x_3 + u_1 + u_2$$

- **Solution: (14 Marks)**

- ✓ **Find state model: (4 Marks)**

From the given system state model is written as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

✓ **Check for controllability (10 Marks)**

Kalman's test for controllability:

$$Q_c = [B \quad AB \quad A^2B]$$

$$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, AB = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -3 & -3 \end{bmatrix}, A^2B = \begin{bmatrix} 1 & 1 \\ -3 & -3 \\ 7 & 7 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 & -3 \\ 1 & 1 & -3 & -3 & 7 & 7 \end{bmatrix}$$

Take any 3X3 sub matrix and find determinant.

$$Q_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -3 \end{bmatrix}$$

 $|Q_c| = 0$ Hence it is not completely controllable.

7. Check the controllability of the system by Kalman's test whose state model is given as,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} u; \quad y = (1 \quad 0 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (\text{Nov/Dec - 16})$$

• **Given: (2 Marks)**

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} u; \quad y = (1 \quad 0 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

• **Solution: (14 Marks)**

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, C = (1 \quad 0 \quad 0)$$

✓ **Step: 1 (7 Marks)**

Kalman's test for controllability:

$$Q_c = [B \quad AB \quad A^2B]$$

$$B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, AB = \begin{bmatrix} 2 \\ -6 \\ 4 \end{bmatrix}, A^2B = \begin{bmatrix} -6 \\ 14 \\ -24 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 2 & -6 \\ 2 & -6 & 14 \\ 0 & 4 & -24 \end{bmatrix}$$

 $|Q_c| = 48 \neq 0$ Hence it is completely controllable.
✓ **Step: 2 (7 Marks)**

Kalman's test for Observability:

$$Q_o = [C^T \quad A^T C^T \quad A^{T^2} C^T]$$

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, A^T = \begin{bmatrix} 0 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 0 & -3 \end{bmatrix}, A^T C^T = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, A^{T^2} C^T = \begin{bmatrix} -2 \\ -3 \\ 0 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$|Q_o| = 0$ Hence it is not observable.

8. Consider the system defined by $\dot{X} = Ax + BU$, $Y = Cx$ where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$;

$C = [10 \ 5 \ 1]$. Check the controllability and observability of the system. (Nov/Dec - 15)

• **Given: (2 Marks)**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; C = [10 \ 5 \ 1]$$

• **Solution: (14 Marks)**

✓ **Step: 1 (7 Marks)**

Kalman's test for controllability:

$$Q_c = [B \ AB \ A^2B]$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, AB = \begin{bmatrix} 1 \\ 1 \\ -12 \end{bmatrix}, A^2B = \begin{bmatrix} 1 \\ -12 \\ 61 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -12 \\ 1 & -12 & 61 \end{bmatrix}$$

$|Q_c| = -96 \neq 0$ Hence it is completely controllable.

✓ **Step: 2 (7 Marks)**

Kalman's test for Observability:

$$Q_o = [C^T \ A^T C^T \ A^{T^2} C^T]$$

$$C^T = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}, A^T C^T = \begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix}, A^{T^2} C^T = \begin{bmatrix} 6 \\ 5 \\ 5 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 10 & -6 & 6 \\ 5 & -1 & 5 \\ 1 & -1 & 5 \end{bmatrix}$$

$|Q_o| = 96 \neq 0$ Hence it is completely observable.

9. Consider a system with state space model given below. $\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 5 \\ -24 \end{bmatrix} u$;

$y = [1 \ 0 \ 0]x + [0]u$; Verify that the system is observable and controllable. (May/June - 15)

• **Given: (2 Marks)**

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 5 \\ -24 \end{bmatrix} u; \quad y = [1 \ 0 \ 0]x + [0]u$$

- **Solution: (14 Marks)**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -1 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 5 \\ -24 \end{bmatrix}; C = [1 \ 0 \ 0]$$

- ✓ **Step: 1 (7 Marks)**

Kalman's test for controllability:

$$Q_c = [B \ AB \ A^2B]$$

$$B = \begin{bmatrix} 0 \\ 5 \\ -24 \end{bmatrix}, AB = \begin{bmatrix} 5 \\ -24 \\ -1 \end{bmatrix}, A^2B = \begin{bmatrix} -24 \\ -1 \\ 116 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 5 & -24 \\ 5 & -24 & -1 \\ -24 & -1 & 116 \end{bmatrix}$$

$$|Q_c| = 11164 \neq 0 \text{ Hence it is completely controllable.}$$

- ✓ **Step: 2 (7 Marks)**

Kalman's test for Observability:

$$Q_o = [C^T \ A^T C^T \ A^{T^2} C^T]$$

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -5 \\ 0 & 1 & -1 \end{bmatrix}, A^T C^T = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, A^{T^2} C^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_o| = 1 \neq 0 \text{ Hence it is completely observable.}$$

10. Test the controllability and observability of the system whose state space representation is

$$\text{given as } \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ -2 & -3 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u; y = [1 \ 0 \ 0] \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} \quad (\text{Nov/Dec - 14})$$

- **Given: (2 Marks)**

$$\begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ -2 & -3 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u \quad y = [1 \ 0 \ 0] \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix}$$

- **Solution: (14 Marks)**

$$A = \begin{bmatrix} 0 & 0 & 2 \\ -2 & -3 & 0 \\ 0 & 2 & 3 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}; C = [1 \ 0 \ 0]$$

- ✓ **Step: 1 (7 Marks)**

Kalman's test for controllability:

$$Q_c = [B \ AB \ A^2B]$$

$$B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, AB = \begin{bmatrix} 0 \\ -6 \\ 4 \end{bmatrix}, A^2B = \begin{bmatrix} 8 \\ 18 \\ 0 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 8 \\ 2 & -6 & 18 \\ 0 & 4 & 0 \end{bmatrix}$$

$|Q_c| = 64 \neq 0$ Hence it is completely controllable.

✓ **Step: 2 (7 Marks)**

Kalman's test for Observability:

$$Q_o = [C^T \quad A^T C^T \quad A^{T^2} C^T]$$

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, A^T = \begin{bmatrix} 0 & -2 & 0 \\ 0 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}, A^T C^T = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, A^{T^2} C^T = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 2 & 6 \end{bmatrix}$$

$|Q_o| = -8 \neq 0$ Hence it is completely observable.

11. Test the controllability and observability of the system whose state space representation is

given as $\begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u; y = [1 \quad 0 \quad 0] \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix}$ (May/Jun - 16)

• **Given: (2 Marks)**

$$\begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u; \quad y = [1 \quad 0 \quad 0] \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix}$$

• **Solution: (14 Marks)**

$$A = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}; C = [1 \quad 0 \quad 0]$$

✓ **Step: 1 (7 Marks)**

Kalman's test for controllability:

$$Q_c = [B \quad AB \quad A^2B]$$

$$B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, AB = \begin{bmatrix} 0 \\ -6 \\ 4 \end{bmatrix}, A^2B = \begin{bmatrix} 4 \\ 18 \\ -24 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 4 \\ 2 & -6 & 18 \\ 0 & 4 & -24 \end{bmatrix}$$

$|Q_c| = 32 \neq 0$ Hence it is completely controllable.

✓ **Step: 2 (7 Marks)**

Kalman's test for Observability:

$$Q_o = [C^T \quad A^T C^T \quad A^{T^2} C^T]$$

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, A^T = \begin{bmatrix} 0 & -2 & 0 \\ 0 & -3 & 2 \\ 1 & 0 & -3 \end{bmatrix}, A^T C^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, A^{T^2} C^T = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

$|Q_o| = 3 \neq 0$ Hence it is completely observable.

12.A system is characterized by the transfer function $\frac{Y(S)}{U(S)} = \frac{3}{S^3 + 6S^2 + 11S + 6}$. Find the state and output equation in matrix form and also test the Controllability and Observability of the system. (Apr/May-17)

- Given: (2 Marks)

$$\frac{Y(S)}{U(S)} = \frac{3}{S^3 + 6S^2 + 11S + 6}$$

- Solution: (14 Marks)

- ✓ Step: 1 (4 Marks)

Write state model equation

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} [u]$$

$$y = [2 \quad 0 \quad 0] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, C = [2 \quad 0 \quad 0]$$

- ✓ Step: 2 (5 Marks)

Kalman's test for controllability:

$$Q_c = [B \quad AB \quad A^2B]$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, AB = \begin{bmatrix} 0 \\ 2 \\ -12 \end{bmatrix}, A^2B = \begin{bmatrix} 2 \\ -12 \\ 72 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & -12 \\ 2 & -12 & 72 \end{bmatrix}$$

$|Q_c| = 8 \neq 0$ Hence it is completely controllable.

- ✓ Step: 3 (5 Marks)

Kalman's test for Observability:

$$Q_o = [C^T \quad A^T C^T \quad A^{T^2} C^T]$$

$$C^T = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}, A^T C^T = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, A^{T^2} C^T = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$|Q_o| = -8 \neq 0$ Hence it is completely observable.

13. A discrete time is described by the difference equation $y(k+2)+5y(k+1)+6y(k)=u(k)$, $y(0)=y(1)=0$; $T=1$ sec, Determine (i) State model in canonical form. (ii) State transition matrix.
(Nov/Dec - 16, Nov/Dec - 14)

- **Given: (2 Marks)**

$$y(k+2)+5y(k+1)+6y(k)=u(k),$$

$$y(0)=y(1)=0; T=1 \text{ sec}$$

- **Solution: (14 Marks)**

- ✓ **Find z-transform: (8 Marks)**

$$z^2Y(z) + 5zY(z) + 6Y(z) = U(z)$$

$$\text{transfer function} = \frac{Y(z)}{U(z)} = \frac{1}{z^2 + 5z + 6}$$

General term,

$$\frac{Y(z)}{U(z)} = \frac{C_1}{z+a_1} + \frac{C_2}{z+a_2}$$

Solve this eqn. by partial fraction technique,

$$\frac{Y(z)}{U(z)} = -\frac{1}{z+3} + \frac{1}{z+2} \quad \text{----- (1)}$$

- ✓ **Find the state model in canonical form: (4 Marks)**

The general state model is given by,

$$\begin{bmatrix} X_1(k+1) \\ X_2(k+1) \end{bmatrix} = \begin{bmatrix} -a_1 & 0 \\ 0 & -a_2 \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u(k)]$$

$$Y(k) = [C_1 \quad C_2] \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + u(k)$$

From the eqn. (1) state model is,

$$\begin{bmatrix} X_1(k+1) \\ X_2(k+1) \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u(k)]$$

$$Y(k) = [-1 \quad 1] \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + u(k)$$

- ✓ **Find the state transition matrix: (4 Marks)**

$$\phi_k = A^k = Z^{-1}\{[zI - A]^{-1}z\}$$

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$[zI - A]^{-1} = \begin{bmatrix} \frac{1}{z+3} & 0 \\ 0 & \frac{1}{z+2} \end{bmatrix}$$

$$\text{state transition matrix } A^k = \begin{bmatrix} (-3)^k & 0 \\ 0 & (-2)^k \end{bmatrix}$$